References for the summer school „Understanding stratified spaces from an analytic and topological viewpoint“

PART 1 (Markus Banagl)

(1) M. Banagl, *Topological Invariants of Stratified Spaces*,
Series: Springer Monographs in Mathematics
2007, XII, 262 pages, 14 illus., with Blank pages: II, XII, 26, 58,
160, 242, 250, 254, 260, 263, 264, Hardcover;
ISBN-10: 3-540-38585-1, ISBN-13: 978-3-540-38585-1
Comment: Introduces sheaf theory and the derived category of constructible complexes of sheaves. Intersection homology is discussed both from the PL and from the sheaf theoretic point of view. The more advanced chapters focus on the signature and L-classes of singular spaces.

(2) M. Banagl, *Intersection Spaces, Spatial Homology Truncation, and String Theory*,
Series: Springer Lecture Notes in Mathematics 1997,
2010, XVI, 217 pages, ISBN: 978-3-642-12588-1
Comment: Introduces intersection spaces and their basic properties. The last chapter discusses the Physics and Mathematics of conifold transitions. For example, it is explained how intersection cohomology and the cohomology of intersection spaces each see different aspects of type IIA and IIB string theory, connected to massless D-branes arising during the formation of singularities. This is illustrated with concrete calculations on the Calabi-Yau quintic.

(3) M. Banagl, L. Maxim, *Deformation of Singularities and the Homology of Intersection Spaces*, J. Topol. Anal. 4 (2012), no. 4.
Comment: This paper proves the Stability Theorem for intersection space cohomology in the context of complex projective hypersurface singularities.

(4) M. Banagl, *The Signature of Singular Spaces and its Refinements to Generalized Homology Theories*, in:
Topology of Stratified Spaces (Friedman, Hunsicker, Libgober, Maxim, eds.),
Mathematical Sciences Research Institute Publications 58,
Cambridge University Press, New York, 2011, 223 - 249.
Comment: An expository paper discussing various bordism theories of singular spaces. Invariants yielding calculations of these theories are described.

(5) M. Banagl, S. E. Cappell, J. L. Shaneson, *Computing twisted signatures and L-classes of stratified spaces*, Math. Ann. 326 (2003), no. 3, 589 - 623.
Comment: The L-class of a singular space, twisted by a globally defined local coefficient system, is computed in terms of the

Chern character of the system's K-theory signature and the
L-class of the underlying space.

(6) A. Borel et al., *Intersection Cohomology*,
Series: Progress in Mathematics, Vol. 50, 1984, Birkhäuser.
Comment: A thorough sheaf theoretic introduction to intersection cohomology. A basic command of sheaf theory is assumed.

(7) M. Goresky, R. D. MacPherson, *Intersection Homology Theory*,
Topology 19 (1980), 135 - 162
Comment: One of the two foundational papers on intersection homology. The theory is introduced using triangulations. Thom's method of defining L-classes is used, together with intersection homology, to define such classes for singular spaces.

(8) M. Goresky, R. D. MacPherson, *Intersection Homology II*,
Invent. Math. 71 (1983), 77 - 129
Comment: The second foundational paper on intersection homology. Here, sheaf theory rather than PL topology is the basis for defining intersection homology in terms of pushforward and truncation functors ("Deligne's formula").

(9) F. Kirwan, J. Woolf, *An introduction to intersection homology theory*,
2nd ed., Chapman & Hall/CRC
Comment: In addition to providing a solid introduction to intersection homology, the book provides insights into many applications. It also discusses L2-cohomology.

(10) Siegel, P. H., Witt spaces: *A geometric cycle theory for KO-homology at odd primes*, Amer. J. Math. 105 (1983), 1067 - 1105
Comment: The paper that introduces the Witt condition on stratified spaces and computes Witt bordism. As the title announces, the relation to ko-homology is investigated as well, using ideas of D. Sullivan.

PART 2 (Jochen Brüning)

(1) Bulletin of the AMS **49** (2012) 4, S. 469-506.
Comment: This volume reproduces two fundamental papers on which the theory of stratified spaces rests, to wit
(1a) René Thom: *Ensembles et morphismes stratifiés,*
(1b) John Mather: *Notes on Topological Stability.*
Comment: Everybody who wants to work with stratified spaces should read these papers and study the one by Mather, since it casts the notions and ideas of Thom into a complete theory with all details. Highly recommended is also the Introduction to the volume by Mark Goresky, which outlines the history of the subject and contains a rich bibliography.

(2) A. Verona: *Stratified Mappings - Structure and Triangulability*, Lecture Notes in Mathematics **1102**, Springer Verlag, 1984. MR771120

(3) M. Pflaum: *Analytic and Geometric Study of Stratified Spaces*, Lecture Notes in Mathematics **1768**, Springer Verlag, 2001. MR1869601
Comment: These excellent treatises describe the state of the theory in great detail, though with a different emphasis, showing their (historical) distance of 20 years.

(3) J. Cheeger: *On the spectral geometry of spaces with cone-like singularities*, Proc. Natl. Acad. Sci. USA **76** (1979), 2103-2106.
(4) J. Cheeger: *On the Hodge theory of Riemannian pseudomanifolds*, Proc. Sympos.Pure Math. **36** (1980), 91-146.
(5) J. Cheeger: *Spectral geometry of singular Riemannian spaces*, J. Differential Geom. **18** (1983), 575-657.
Comment: These papers develop the essential parts of Geometric Analysis and the techniques to treat them in the case of stratified spaces: the condition for the essential self-adjointness of the de Rham-Hodge operator, and the computation of the L^2-cohomology of metric cones and the expansion of the heat trace of the Laplacian in this case. Even though Cheeger deals only with (certain) finite simplicial complexes, he has made the essential discovery that there are different closed extensions of the de Rham operator which lead to Hilbert complexes. This fact appears e. g. as an obstacle for the existence of a signature operator and also leads to various different L^2-cohomology theories.

(6) J. Brüning, R.T. Seeley: *Regular singular asymptotics*, Advances in Math. **58** (1985), 133-148.
Comment: This paper presents an asymptotic expansion lemma which is adapted to conic metrics and has proved very useful in the analysis of the heat trace.

(7) J. Brüning, R.T. Seeley: *The resolvent extension for second order regular singular operators*, J. Funct. Anal. **73** (1987), 369-429.
Comment: In this paper, the resolvent of the Laplace operator on a cone is constructed and analysed, and the methods of (6) are used to derive the asymptotic extension of the heat trace of the Laplacian.

(8) J. Brüning, R.T. Seeley: *An index theorem for first order regular singular operators*, Am. J. Math. **110** (1988), 659-714.
Comment: In this paper, the selfadjoint extensions of the de Rham-Hodge operator are analysed and classified in the case of isolated conic singularities. A careful spectral analysis is used to derive index formulas for the Gauss-Bonnet and the signature operator, much in the spirit of Cheeger (5).

(9)J. Brüning, R.T. Seeley: *The expansion of the resolvent near a singular stratum of conical type*, J. Funct. Anal. **95** (1991), 255-290.
Comment: In this paper, the methods developed before are extended to families of cones over a compact base, with certain limitations. Notably, only the Friedrichs extension of the Laplacian is considered. Therefore, conclusions for the de Rham-Hodge operator and its index formulas are limited to the case of essential selfadjointness, but the index formulas are not yet carried out. This is done in the paper
(10) J. Brüning: *The signature operator on manifolds with a conical singular stratum*, Astérisque **328** (2009), 1-44.

(11) Brasselet, J.-P.; Hector, G.; Saralegi, M.: *Théorème de de Rham pour les variétés stratifiées*. [The de Rham theorem for stratified manifolds], Ann. Global Anal. Geom. **9** (1991) 3, 211-243.
(12) Brasselet, J.-P.; Hector, G.; Saralegi, M.: *L^2-cohomologie des espaces stratifiés*, Manuscripta Math. **76** (1992) 1, 21-32.

Comment: The de Rham operator on arbitrary stratified spaces and the de Rham-Hodge operator for such spaces equipped with an "adapted" or "conic" metric have been dealt with in these important papers by Brasselet and his coworkers; in particular, they give a transparent proof for the existence of such metrics which had been obtained before in special cases by Nagase.

(13) Albin, Pierre; Leichtnam, Éric; Mazzeo, Rafe; Piazza, Paolo: *The signature package on Witt spaces*. Ann. Sci. Éc. Norm. Supér. (4) **45** (2012), no. 2, 241–310. MR2977620
Comment: Recently, Albin et al. have undertaken a thorough study of the signature operator on stratified spaces using different methods which derive from the work of Melrose and hence consider stratified spaces as resolved by manifolds with corners and involves various calculi for (pseudo)differential operators. On Witt spaces, where the de Rham-Hodge operator can be assumed to be essentially self-adjoint, the authors give the construction of the signature operator with coefficients in a Hilbert bundle in the spirit of Mishchenko, a point of view we have not included in our discussions yet.