The Problem Sheets are a tool to show first of all what should be mastered from the material in the lecture; these are the homework problems marked with “P”. Moreover, I will indicate certain aspects of the theory which are desirable to know but would lead too far if presented in the lecture; such areas are marked with “S” (for seminar). You are invited to present such an area (or part of it) in a seminar talk, possibly together with other participants.

You need not hand in written solutions, your admission to the examination will be based on your activity in the seminar part added to the lecture, which will normally take place on Tuesday (11:00-12:30, Rudower Chaussee 25, 2.009). The examination could either be an oral one of 30 minutes duration, or a “Hausarbeit”, i.e. a carefully written version of a seminar presentation or of another topic close to the material of the lecture on which we have to agree; your own suggestions are welcome.

Humboldt-Universität zu Berlin
Institut für Mathematik
Prof. Dr. Jochen Brüning
Lecture **The Atiyah-Singer Index Theorem**, WS 2011/12

Exercises no. 2

due Nov 8, 2011

1.P Give a detailed proof of the Isomorphism Theorem stated in the lecture of Nov. 2.

2.P Show that the isomorphism class of a principal G-bundle can in fact be described by a G-cocycle.

3.P Show that the principal bundle P_G, as derived from a coordinate bundle $(M, \pi, B, F, \mathcal{A}, g^G_{\mathcal{A}})$, furnishes an isomorphism between

$$P_G \times_\rho F \qquad \text{and} \qquad \beta = (M, \pi, B, F).$$

(We say that "β has the *structure group* G" if, for some topological subgroup G of $\mathrm{Homeo}(F)$ and some fibration atlas $\mathcal{A}$, β admits a cocycle $g^G_{\mathcal{A}}$ with values in G.)

4.P Show that the map $\breve{\pi} : S^3 \ni z \mapsto [z] \in \mathbb{C}P^1$ induces a smooth fibration (the *Hopf fibration*)

$$S^1 \hookrightarrow S^3 \xrightarrow{\pi} S^2.$$

Show also that the curvature tensor R_π does not vanish.