1. Let \( U_n, n \in \mathbb{N} \), be an exhaustion of \( M \) by open subsets. Prove that for all \( 0 \leq f \in L^2(M) \), \( t > 0 \), one has
\[
\| P_t f \|_{L^2(U_n)} \leq C_n \| f \|_{L^2(M)}
\]
\( \mu \)-a.e.

2. Let \( \zeta : I \times M \to \mathbb{R} \) be continuous (with some interval \( I \subset \mathbb{R} \)), and assume \( \partial_t \zeta \) exists (classically) and is continuous on \( I \times M \). Show that for all open relatively compact \( U \subset M \), the Banach-space valued map
\[
I \ni t \mapsto \zeta(t, \cdot) \in C_b(U)
\]
is strongly differentiable, and that its strong derivative \( (d/dt) \zeta \) equals \( \partial_t \zeta \) on \( I \times U \).