Topology II

Exercise sheet 10

Exercise 1.
Show that the short exact sequence from the universal coefficient theorem does **not** split natural.
Hint: Consider the projection map

$$f\colon \mathbb{R}P^2 \cong D^2 \cup \mathbb{R}P^1 \longrightarrow \mathbb{R}P^2/\mathbb{R}P^1 \cong S^2.$$

Exercise 2.

(a) Compute the homology of the n-torus $T^n := S^1 \times \cdots \times S^1$.

(b) Let M and N be closed **topological** manifolds. Show that $M \times N$ is orientable if and only if M and N are orientable.

Exercise 3.

(a) Let $p\colon X' \to X$ be a covering of a connected CW-complex X. Describe a CW-structure on X' such that p is cellular and X' has the same dimension as X.

(b) Assume that X is a finite CW-complex and p a covering of finite degree. Compute the Euler characteristic of X' from the Euler characteristic of X.

Exercise 4.
A map $f\colon X \to Y$ induces an isomorphism on homology with $\mathbb{Z}$-coefficients if and only if f induces an isomorphism on homology with $\mathbb{Q}$-coeffcients and $\mathbb{Z}_p$-coeffcients for all primes p.

Bonus exercise 1.
$\operatorname{Tor}(A, \mathbb{Q}/\mathbb{Z})$ is isomorphic to the torsion subgroup of A.

Bonus exercise 2.
$\mathbb{R}P^3$ is homeomorphic to $SO(3)$.

Bonus exercise 3.

(a) Describe the persistent homology (by drawing its barcode diagram) of the filtration on the Christmas star in Figure 1 induced by its height function.

(b) Express the persistent homology groups of a sequence of subcomplexes K_n in terms of cycles and boundaries of the subcomplexes K_n.

(c) Describe an algorithm to compute the homology groups of a simplicial complex with $\mathbb{Z}$- and $\mathbb{Z}_2$-coefficients.

(d) Describe an algorithm to compute the persistent homology groups of a sequence of subcomplexes.

(e) Estimate the runtime of your algorithms in (c) and (d).

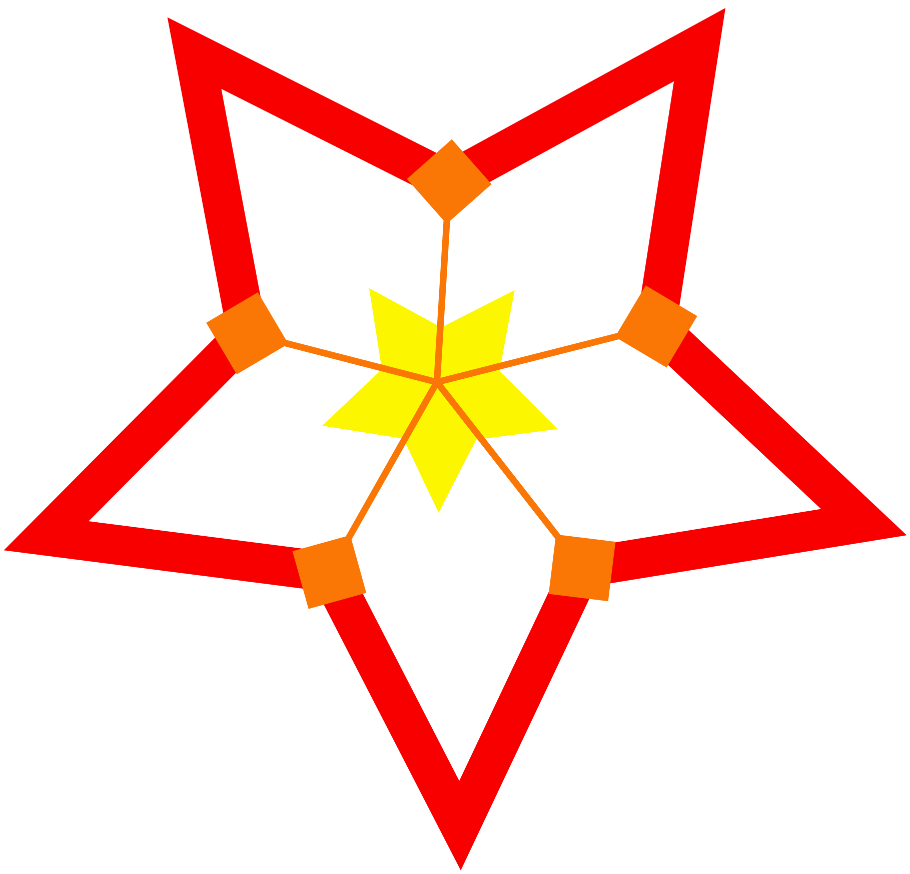

Abbildung 1: A Christmas star.

This sheet will be discussed on Friday 10.1. and should be solved by then.