Exercise 1.
Show that the short exact sequence from the universal coefficient theorem does not split natural.

Hint: Consider the projection map

\[ f : \mathbb{R}P^2 \cong D^2 \cup \mathbb{R}P^1 \to \mathbb{R}P^2 / \mathbb{R}P^1 \cong S^2. \]

Exercise 2.

(a) Compute the homology of the \( n \)-torus \( T^n := S^1 \times \cdots \times S^1 \).

(b) Let \( M \) and \( N \) be closed topological manifolds. Show that \( M \times N \) is orientable if and only if \( M \) and \( N \) are orientable.

Exercise 3.

(a) Let \( p : X' \to X \) be a covering of a connected CW-complex \( X \). Describe a CW-structure on \( X' \) such that \( p \) is cellular and \( X' \) has the same dimension as \( X \).

(b) Assume that \( X \) is a finite CW-complex and \( p \) a covering of finite degree. Compute the Euler characteristic of \( X' \) from the Euler characteristic of \( X \).

Exercise 4.

A map \( f : X \to Y \) induces an isomorphism on homology with \( \mathbb{Z} \)-coefficients if and only if \( f \) induces an isomorphism on homology with \( \mathbb{Q} \)-coefficients and \( \mathbb{Z}_p \)-coefficients for all primes \( p \).

Bonus exercise 1.
Tor(\( A, \mathbb{Q}/\mathbb{Z} \)) is isomorphic to the torsion subgroup of \( A \).

Bonus exercise 2.
\( \mathbb{R}P^3 \) is homeomorphic to \( SO(3) \).
**Bonus exercise 3.**

(a) Describe the persistent homology (by drawing its barcode diagram) of the filtration on the Christmas star in Figure 1 induced by its height function.

(b) Express the persistent homology groups of a sequence of subcomplexes $K_n$ in terms of cycles and boundaries of the subcomplexes $K_n$.

(c) Describe an algorithm to compute the homology groups of a simplicial complex with $\mathbb{Z}$- and $\mathbb{Z}_2$-coefficients.

(d) Describe an algorithm to compute the persistent homology groups of a sequence of subcomplexes.

(e) Estimate the runtime of your algorithms in (c) and (d).

![Christmas star](image-url)

**Abbildung 1: A Christmas star.**

This sheet will be discussed on Friday 10.1. and should be solved by then.