WS 2019/2020 Marc Kegel

Topology II

Exercise sheet 3

Exercise 1.
Let X be the topological space consisting of a single point. Compute all homology groups of X directly from the definitions. For which other spaces can we compute all homology groups?

Exercise 2.
Find a way to relate the homology group of a topological space X to the homology groups of its path-connected components.

Exercise 3.
Fill in the details of the proof of Theorem 3.5. from the lecture and draw a 2- and a 3-dimensional picture visualizing it.

Exercise 4.
Let X be a path-connected space. Show that $H_0(X) \cong \mathbb{Z}$ and that $H_1(X)$ is isomorphic to the abelization $\pi_1^{ab}(X)$ of the fundamental group of X.
Hint: Try to adapt the arguments for simplicial homology from last semester to singular homology.

This sheet will be discussed on Friday 8.11. and should be solved by then.