WS 2019/2020 Marc Kegel

Topology II

Exercise sheet 5

Exercise 1.
We denote by A and B the curves on the surface Σ_2 of genus 2 shown in Figure 1. Compute the relative homology groups $H_k(\Sigma_2, A)$ and $H_k(\Sigma_2, B)$.

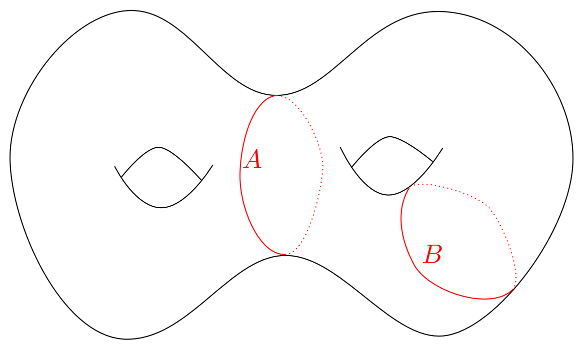

Abbildung 1: Two curves A and B on a genus 2-surface

Exercise 2.

(a) $T^n := S^1 \times \cdots S^1$ admits the structure of a CW-complex.

(b) Any closed surface admits the structure of a CW-complex.

(c) Describe a CW-structure on Σ_2 such that A and B shown in Figure 1 are subcomplexes.

(d) Describe CW-complexes on $\Sigma_g \times \Sigma_h$.
Hint: Use (b) and the technique from (a).

Exercise 3.
Let X be a finite connected CW-complex. How can the fundamental group of X be computed?
Bonus: What can be said if X is **not** finite?

Exercise 4.
Prove the invariance of the domain:
Let $U, V \subset S^n$ be homeomorphic. Then U is open if and only if V is open.
Hint: Use Proposition 3.21 and Theorem 3.17.

Bonus exercise 1.
Do the simplicial homology groups (discussed in last semesters course) fulfill the Eilenberg–Steenrod axioms?

Bonus exercise 2.

(a) Describe explicitly a sequence of embeddings $f_k \colon D^3 \to S^3$ converging to an embedding $f \colon D^3 \to S^3$, such that $f(S^2)$ is homeomorphic to Alexander's horned sphere.

(b) Show that the complement of $f(D^3)$ is not contractible by showing that it admits a non-vanishing element γ in $\pi_1(S^3 \setminus f(D^3))$.
Bonus: What is the fundamental group of its complement?

(c) Show explicitly that γ is nullhomologous. (Explicitly means here without using Proposition 3.21.)

(d) Can we construct an embedding of S^2 into S^3 such that both components of the complement are not contractible?

Hint: For some help in this exercise one could have a look at page 170 in Hatcher's book on algebraic topology.

This sheet will be discussed on Friday 22.11. and should be solved by then.