WS 2019/2020 Marc Kegel

Topology II

Exercise sheet 8

Exercise 1.

(a) Let X be a connected 1-dimensional CW-complex. Show $\pi_n(X) = 0$ for all $n \geq 2$.

(b) Compute all homotopy groups of surfaces Σ_g of genus $g \geq 1$ by applying Hurewitcz's theorem to its universal covering.

Bonus: Let F be a surface (not necessarily compact) with infinite fundamental group. Compute its higher homotopy groups.

Exercise 2.
Compute the second homotopy groups of $\mathbb{C}P^n$ and $S^1 \vee S^2$.

Exercise 3.
Let $f\colon (S^n, N) \to (S^n, N)$ be a homeomorphism of S^n which preserves the north pole N. Which element represents f in $\pi_n(S^n, N)$?

Exercise 4.
A connected topological space X with only one non-vanishing homotopy group $\pi_n(X) \cong G$ is called **Eilenberg–MacLane space** $K(G, n)$.

(a) Construct an Eilenberg–MacLane space for arbitrary G and n (assuming G to be abelian if $n \geq 1$).
Hint: It may be helpful to have a look at Exercise 3 from Sheet 7.

(b) Let $G_1, G_2, \ldots$ be a (possible infinite) sequence of (not necessarily finitely presented) groups (abelian for $n \neq 1$). Construct a connected CW-complex X with homotopy groups

$$\pi_k(X) \cong G_k.$$

Bonus: When is such a space unique up to homotopy?

This sheet will be discussed on Friday 13.12. and should be solved by then.