WS 2020/2021 Marc Kegel

Topology II

Exercise sheet 5

Exercise 1.
Compute the homology groups of all closed surfaces using the Mayer–Vietoris sequence.
Bonus: Do the same for all compact surfaces.

Exercise 2.
We denote by A and B the curves on the surface Σ_2 of genus 2 shown in Figure 1. Compute the relative homology groups $H_k(\Sigma_2, A)$ and $H_k(\Sigma_2, B)$.

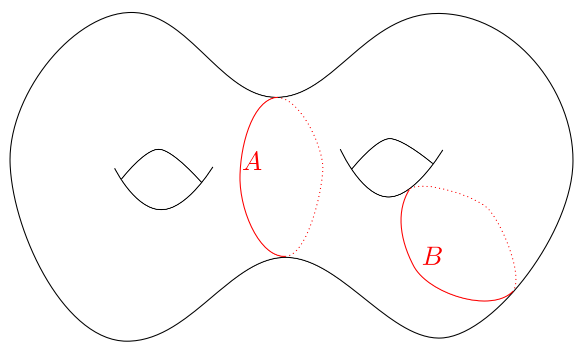

Abbildung 1: Two curves A and B on a genus 2-surface

Exercise 3.
Let K be a knot in S^3, i.e. the image of an embedding of S^1 into S^3. Compute all homology groups of $S^3 \setminus K$.
Bonus: What can you say about the homotopy groups of $S^3 \setminus K$?

Exercise 4.
Prove the invariance of the domain:
Let $U, V \subset S^n$ be homeomorphic. Then U is open if and only if V is open.
Hint: Use Proposition 3.21 and Theorem 3.17.

p.t.o.

Bonus exercise.

(a) Describe explicitly a sequence of embeddings $f_k\colon D^3 \to S^3$ converging to an embedding $f\colon D^3 \to S^3$, such that $f(S^2)$ is homeomorphic to Alexander's horned sphere.

(b) Show that the complement of $f(D^3)$ is not contractible by showing that it admits a non-vanishing element γ in $\pi_1(S^3 \setminus f(D^3))$.
Bonus: What is the fundamental group of its complement?

(c) Show explicitly that γ is nullhomologous. (Explicitly means here without using Proposition 3.21.)

(d) Can we construct an embedding of S^2 into S^3 such that both components of the complement are not contractible?

Hint: For some help in this exercise one could have a look at page 170 in Hatcher's book on algebraic topology.

This sheet will be discussed on Friday 4.12. and should be solved by then.