Marc Kegel
Shubham Dwivedi

Topology II
Exercise sheet 5

Exercise 1.
Finish the prove of Theorem 3.7 (i.e. verify the exactness of the long exact sequence) and deduce from it Corollary 3.8.

Exercise 2.
Let S^n be the unit sphere in $\mathbb{R}^{n+1}$ and identify S^{n-1} with the equator sphere $\{x \in S^n | x_{n+1} = 0\}$ of S^n. We denote by $D_\pm^n = \{x \in S^n | \pm x_{n+1} \geq 0\}$ the upper (lower) hemisphere of S^n. Create sketches in low dimensions visualizing this situation and compute the singular homology groups of S^n, (D^{n+1}, S^n), and (S^n, D_+^n).

Exercise 3.
We denote by A and B the curves on the surface Σ_2 of genus 2 shown in Figure 1. Compute the relative homology groups $H_k(\Sigma_2, A)$ and $H_k(\Sigma_2, B)$.
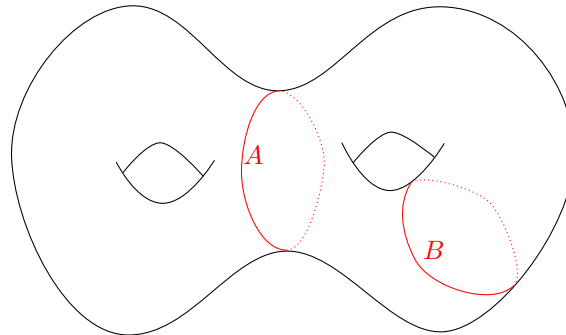

Abbildung 1: Two curves A and B on a genus 2-surface

Bonus exercise.
Fill in the details of the proof of Theorem 3.5. from the lecture and draw a 2- and a 3-dimensional picture visualizing it.

This sheet will be discussed on Wednesday 24.11. and should be solved by then.