

Differentialgeometrie I

Exercise sheet 5

Exercise 1.

Compute the Gaussian curvature of

$$\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\}$$

and verify that it is negative everywhere.

Exercise 2.

- (a) Let M be a surface such that each of its points is an umbilical point. Then each point of M has a neighborhood in M which is an open subset of a plane or a sphere.
- (b) If $g_{12} \equiv L_{12} \equiv 0$ then the u_i curves are curvature lines.
- (c) Outside of umbilical points the converse of (b) holds. What happens in umbilical points?
- (d) Latitude circles and meridians of a rotation surface are curvature lines.

Exercise 3.

Consider the surface given by the parametrization

$$x(u, v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2 \right).$$

Compute its first and second fundamental form, curvatures, and curvature lines.