WS 2023/24 Marc Kegel

Contact Geometry

Exercise sheet 4

Exercise 1.

(a) Show that the local modifications of front projections depicted in Figure 1 induce isotopies of Legendrian knots.
Remark: The Legendrian-Reidemeister theorem says that in fact two front projections describe Legendrian isotopic knots if and only if the front projections can be transformed into each other via finitely many planar isotopies and the moves from Figure 1.

(b) Show that the pairs of front projections shown in Figures 2 describe isotopic Legendrian knots.

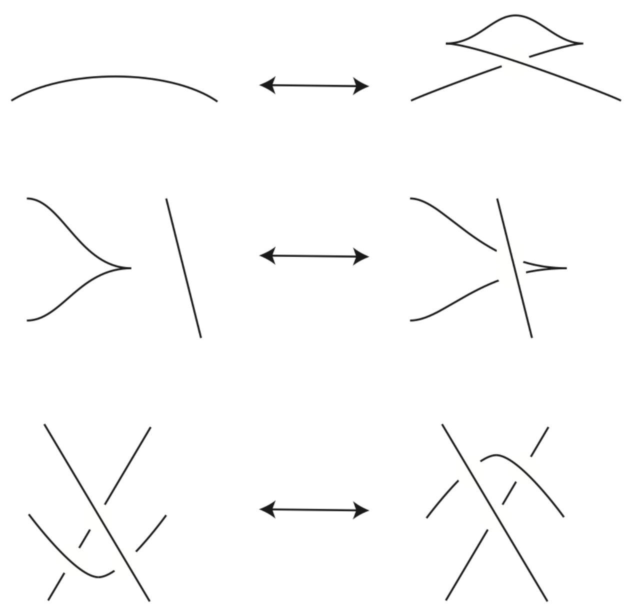

Abbildung 1: Together with the π-rotations around all coordinate axes these local modifications are the Legendrian Reidemeister moves.

Abbildung 2: Front projections of Legendrian unknots (left) and Legendrian figure eight knots (right).

Exercise 2.

(a) Describe transverse realizations of unknots, right- and left-handed trefoils, and figure eight knots.

(b) Describe an algorithm to construct a front projection of the transverse push-off of a Legendrian knot K from a front projection of K.

(c) Discus front projections of links consisting of Legenrian and transverse knots.

Exercise 3.

(a) Fill in the details in the argument from the lecture that the Alexander polynomial is a knot invariant.

(b) Compute the Alexander polynomial of the unknot, the trefoil, and the figure eight knot and deduce that these knots are pairwise non-isotopic.

(c) Show that the figure eight knot has genus 1.

(d) Construct for every natural number $g \in \mathbb{N}_0$ a knot K_g with genus g.

Exercise 4.
The **Lagrangian** projection is the projection to the (x, y)-plane.

(a) The Lagrangian projection of a Legendrian knot K determines K up to isotopy of Legendrian knots (in fact up to translation in z-direction).

(b) Discuss Lagrangian projections of Legendrian knots in analogy to the front projections.

(c) Use the Lagrangian projection to reprove that any smooth knot can be approximated by a Legendrian knot.

(d) Describe an algorithm to construct a Lagrangian projection of a Legendrian knot K from a front projection of K.

Bonus exercise.
Let K be a knot in a 3-manifold M.

(a) Compute the homology groups of the complement of K in M.

(b) Show that K bounds a Seifert surface if and only if K is nullhomologous.
Hint: Try to generalize the proofs from the lecture for knots in S^3.

Bonus exercise.
Prove Theorems 3.1 and 3.2 from the lecture.

This sheet will be discussed on Wednesday 22.11. and should be solved by then.