WS 2023/24 Marc Kegel

Contact Geometry

Exercise sheet 5

Exercise 1.

(a) Prove Lemma 3.12 from the lecture.

(b) Compute the classical invariants of the Legendrian knots from Sheet 4.

(c) Verify that the classical invariants of Legendrian knots stay the same under the Legendrian Reidemeister moves (see Sheet 4).

Exercise 2.

(a) Show that $\mathrm{tb}(K) + \mathrm{rot}(K)$ is for any Legendrian knot K in $(\mathbb{R}^3, \xi_{st})$ an odd number.

(b) Show that any odd number is realized as $\mathrm{tb}(K) + \mathrm{rot}(K)$ for some Legendrian knot K.

Exercise 3.

(a) Show that the stabilization of a Legendrian knot (as defined in the lecture) is a well-defined operation.

(b) Any two Legendrian knots become Legendrian isotopic after sufficiently many stabilizations. *Hint:* Use the Reidemeister theorem for smooth knots.

Exercise 4.

(a) Fill in the details in the argument from the lecture that the Alexander polynomial is a knot invariant.

(b) Compute the Alexander polynomial of the unknot, the trefoil, and the figure eight knot and deduce that these knots are pairwise non-isotopic.

(c) Show that the figure eight knot has genus 1.

(d) Construct for every natural number $g \in \mathbb{N}_0$ a knot K_g with genus g.

(e) Verify that the combinatorial formula for the linking number is preserved under the smooth Reidemeister moves.

Exercise 5.
Describe formulas for computing the Thurston–Bennequin invariant and the rotation number from a Lagrangian projection (see Sheet 4).

This sheet will be discussed on Wednesday 29.11. and should be solved by then.