Holomorphic torsion and Arakelov geometry

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- **Holomorphic torsion**

  Holomorphic torsion is a linear combination of regularized determinants of Kodaira-Laplace operators associated to an Hermitian holomorphic vector bundle over a complex manifold. Its main application is to serve as the main ingredient of a direct image functor in a K_0-theory of vector bundles with Hermitian metrics. The definitions of torsion, torsion forms, equivariant torsion and some examples shall be explained in this talk.

- **Arakelov K-theory**

  For a variety X defined over the primes Spec $\mathbf{Z}$ consider its fiber $X_{\mathbf{C}}$ over the complex numbers. Gillet-Soulé's Arakelov K_0-theory $\hat{K}(X)$ consists of arithmetic vector bundles over X equipped with an Hermitian metric over the Kähler manifold $X_{\mathbf{C}}$, plus certain classes of differential forms. The holomorphic torsion is the main ingredient in the construction of a map $\pi_! : \hat{K}(X) \to \hat{K}(Y)$ for proper morphisms $\pi : X \to Y$. It verifies a Riemann-Roch relation with direct images in Arakelov intersection theory. If certain group schemes act on X and Y, the direct image can be shown to localize on the fixed point subscheme, which can simplify its computation.

- **Applications of the Lefschetz fixed point formula**

  Several application of the fixed point localization of direct images in Arakelov K_0-theory shall be described, e.g. a Bott residue formula for arithmetic characteristic classes, a proof for the Jantzen sum formula classifying representations of Chevalley groups, simple formulae for the (global arithmetic) height of generalized flag spaces, a proof of (a weak form of) the period conjecture by Gross-Deligne, an analogue of Hirzebruch's proportionality principle in Arakelov geometry and other results. A quaternionic torsion for quaternionic Kähler manifolds and its relation with the holomorphic one shall be explained.