Lecture Series at the Summer School 2006

1. Andreas Juhl: *Introduction to Q-curvature and its applications*
2. Janko Latschev: *Holomorphic curves in symplectic geometry*
3. Alan Reid : *The geometry, topology and arithmetic of low-dimensional hyperbolic manifolds*
4. Viktor Schröder: *Large scale geometry of metric spaces*

Introduction to Q-curvature and its applications

Andreas Juhl (Univ. Uppsala/HU Berlin)

The notion of Q-curvature has been introduced by Thomas Branson almost 15 years ago in connection with variational formulas for the determinant of conformally covariant differential operators. It is an intrinsic curvature quantity of any pseudo-Riemannian manifold of even dimension. It has the remarkable property that under conformal changes of the metric its change is described by a conformally covariant *linear* differential operator of high order which generalizes the Yamabe operator. The higher the dimension the more mysterious the Q-curvature. In any dimension the total Q-curvature is related to the conformal anomaly of the renormalization of the Einstein-Hilbert action on asymptotically hyperbolic spaces. The latter renormalization is central in the AdS/CFT duality. Understanding the structure, the geometric significance and the physical meaning of Q-curvature covers a wide area of research. The lectures will be an introduction to this active area.

Holomorphic Curves in Symplectic Geometry

Janko Latschev (LMU München/HU Berlin)

Since their introduction by M. Gromov (pseudo)holomorphic curves are the central tool in symplectic geometry. The invariants which are defined using them (Gromov–Witten invariants, Floer homology, Symplectic Field Theory) are themselves interesting objects to study. The lectures will give a first introduction into this rich theory. We intend to give an overview on recent research at the end. Geometric constructions as well as the algebraic structures imposed by them will be the main theme of the program. We will also address the problems which have to be solved.

The geometry, topology and arithmetic of low-dimensional hyperbolic manifolds

Alan Reid (Univ. Texas)

These lectures will discuss the interaction of the geometry and topology of hyperbolic manifolds in dimensions 2 and 3 with number theory and algebra. We will discuss some basic examples like the modular group, and develop some background in hyperbolic geometry, discrete groups and number theory. The collection of arithmetic hyperbolic manifolds will be studied and we will attempt to describe the interplay between the topology and the number theory of these examples by discussing work motivated by the virtual Haken conjecture for 3-manifolds. We will discuss the interplay of geometry and number theory by studying the structure of the set of closed geodesics of these manifolds.

Large scale geometry of metric spaces

Viktor Schröder (Univ. Zürich)

One motivation to study the large scale geometry of metric spaces comes from geometric group theory. The Cayley graph of a group gives a geometric description of the group, however this graph depends on the particular set of generators. For two different generating sets the corresponding graphs are "quasi-isometric". Thus a quasi-isometry invariant of the Cayley graph is an invariant of the group. Therefore it is useful to develop a theory of metric spaces focussing on quasi-isometry or other large scale invariants. Such a theory developed in the last twenty years. An important concept is the notion of Gromov hyperbolicity which generalizes properties of the standard hyperbolic space to the large scale setting. The lectures give an introduction to the theory of Gromov hyperbolic spaces and some other concepts of large scale geometry as the asymptotic dimension.