Übungen zum Kurs Lorentzgeometrie und Relativitätstheorie

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Übungsblatt 6

Exercise 1: Geodesics

Calculate the geodesic equations in $(\mathbb{R} \times N, \pm dt^2 + f^2(t) \cdot h)$ for (N, h) being a Riemannian metric. Show that in the example $(N, h) = (\mathbb{S}^1, dx^2)$ we get a constant $c \in \mathbb{R}$ such that for all geodesics we have $t'' = 2cf'(t)/f(t)$. Apply this to the examples (for the sign $+$ above) of Euclidan space ($f(t) = t$), the round sphere ($f(t) = \sin(t)$) and hyperbolic space ($f(t) = \sinh(t)$).

Exercise 2: Semicontinuity of d

Show the remaining statement of Theorem 2.39: Assume that (M, g) is globally hyperbolic and prove upper semicontinuity of d^g. **Hint:** Use Geroch's compactness statement and upper semicontinuity of the length function for generalized causal curves.

Exercise 3: Einstein equation

Let us examine when a Pseudo-Riemannian product $(\mathbb{R} \times N, -dt^2 + g_N)$ satisfies the Einstein equation $\mathrm{ric}^g = \Lambda g$. Show first that for $\Lambda = 0$, g is Λ-Einstein if and only if g_N is Ricci-flat. Can you give a corresponding condition for $\Lambda \neq 0$? **Hint:** Show first that the Riemannian curvature tensor factorizes.