Übungen zur Lorentzgeometrie und Mathematischen Relativitätstheorie

Humboldt-Universität zu Berlin, Wintersemester 2018-19
PD Dr.habil. Olaf Müller
Übungsblatt 8

Exercise1: Rigidity of Killing vector fields

Let (M, g) be a semi-Riemannian manifold and let X be a g-Killing vector field on M.

1. Let f be a smooth function on M. Show that fX is Killing if and only if f is constant.

2. Let $S \subset M$ be g-totally geodesic. Show that, for P being the orthogonal projection to TS, $P(X)$ is a g-Killing vector field as well.

Exercise 2: Einstein tensor is divergence-free

Show that $\mathrm{div}(\mathrm{Ein}) = 0$. **Hint:** Contractions of the Bianchi identities may be helpful.

Exercise 3: Equations in the Jebsen theorem

Show that the last equation in the proof of the Jebsen theorem follows from the other three using the Bianchi identity.