

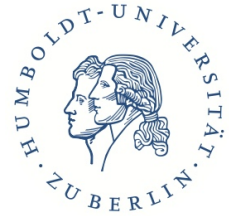
# Übungen zur Lorentzgeometrie und Mathematischen Relativitätstheorie

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## Übungsblatt 8

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### Exercise 1: Rigidity of Killing vector fields

Let  $(M, g)$  be a semi-Riemannian manifold and let  $X$  be a  $g$ -Killing vector field on  $M$ .

1. Let  $f$  be a smooth function on  $M$ . Show that  $fX$  is Killing if and only if  $f$  is constant.
2. Let  $S \subset M$  be  $g$ -totally geodesic. Show that, for  $P$  being the orthogonal projection to  $TS$ ,  $P(X)$  is a  $g$ -Killing vector field as well.

### Exercise 2: Einstein tensor is divergence-free

Show that  $\operatorname{div}(\operatorname{Ein}) = 0$ . **Hint:** Contractions of the Bianchi identities may be helpful.

### Exercise 3: Equations in the Jepsen theorem

Show that the last equation in the proof of the Jepsen theorem follows from the other three using the Bianchi identity.