Mean-risk optimization of electricity portfolios

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We present a mathematical model with stochastic input data for mean-risk optimization of electricity portfolios containing several physical components and energy derivative products. The model is designed for the optimization horizon of one year in hourly discretization. The aim consists in maximizing the mean book value of the portfolio at the end of the optimization horizon and, at the same time, in minimizing the risk of the portfolio decisions. The risk is measured by the conditional value-at-risk and by some multiperiod extension of CVaR, respectively.We present numerical results for a large-scale realistic problem adapted to a municipal power utility and study the effects of varying weighting of risk.

1 Introduction

The deregulation of energy markets has led to an increased awareness of the need for profit maximization with simultaneous consideration of risk management, adapted to individual risk aversion of market participants. Mathematical modelling of such problems with uncertain input data results in mixed-integer large-scale stochastic programming models. We refer to a wide range of literature dealing with power management in a hydro-thermal system and simultaneous optimization of power production and electricity trading, e.g. [7] and [10]. We suppose that each historical observation of electrical load and spot price is a realization of certain bivariate random variables. The joint distribution of the stochastic process will be characterized by a time series model. To ensure the numerical tractability of the optimization problem we generate a large number of Monte-Carlo scenarios from this time series model. By means of a recursive scenario reduction procedure, cf. [6], we generate from this initial approximation of the underlying probability distribution a specific form of an approximation - a scenario tree taking into account the information structure of the optimization problem. For the mathematical description of scenario trees, see

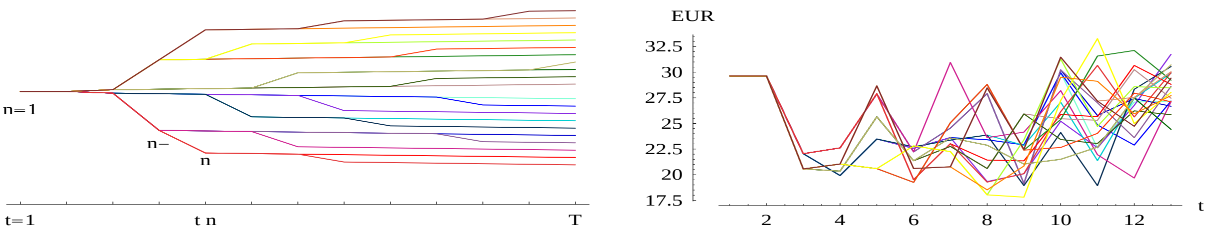

Fig. 1 Scenario tree structure Monthly mean spot prices

Fig.1, we serially number the nodes of the tree. Except for the root node $n = 1$, every node $n \in \mathcal{N}$ has a unique predecessor $n-$ but possibly several successors forming the set $\mathcal{N}_+(n)$. By $\mathcal{N}_T$ we denote the set of leafs of the scenario tree. Further let $path(n)$ be the set of nodes from the root node to the node n, whereas $t(n)$ denotes the time period related to n. A unique node probability π_n is assigned to every node n by setting $\pi_n = \sum_{l\in\mathcal{N}_+(n)} \pi_l$ and π_n for $n \in \mathcal{N}_T$ being the scenario probabilities.

2 Modelling stochastic processes

Based on earlier studies, e.g. [7], and [10] we suggest a decomposition strategy for the original load and spot price series $\{(L_t, C_t^{Sp}),\ t = 1, \ldots, T\}$ in hourly discretization and a separate modelling of intra-daily behavior and average daily behavior. We generate intra-daily scenarios by using a distribution-free resampling procedure based on a cluster analysis. For the description of modelling the bivariate average daily process let L_k denote the load and C_k^{Sp} the spot price at day k. The model comprises a deterministic function $(f^1(k), f^2(k))$ representing a yearly trend with seasonal patterns, a stochastic component $(X^1(k), X^2(k))$ modelled by a bivariate autoregressive moving-average process, whereas extreme spot price outliers

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are modelled by a discretized jump-diffusion process O_k^2 with time-varying jump parameters. For details we refer to [8].

$$\begin{pmatrix} L_k \\ C_k^{Sp} \end{pmatrix} = \begin{pmatrix} f^1(k) \\ f^2(k) \end{pmatrix} + \begin{pmatrix} X_k^1 \\ X_k^2 \end{pmatrix} + \begin{pmatrix} 0 \\ O_k^2 \end{pmatrix}, \quad k = 1, \dots, K = T/24 \tag{1}$$

In addition to physical components the electricity portfolio contains energy derivative products represented by EEX futures of various load and delivery types. Futures are standardized forward transactions. Both buyer and seller agree on the current date to call-off respectively to supply a certain quantity of electricity at a delivery period in the future for a certain price or to effect respective payments. In order to generate arbitrage free future prices as an input for the optimization model we calculate fair prices, adjusted to EEX rules. The settlement price for month futures on the last trading day is the mean spot price for the delivery month and the associated load type. Disregarding transaction fees the fair price for a base load month future at time t with delivery period $\langle T_1, T_2 \rangle$ in a liquid market related to the underlying spot price C_t^{Sp} is defined as:

$$C_{t,T_2}^{f,M} \quad = \quad \mathbb{E}\left(\sum_{j=T_1}^{T_2} C_j^{Sp} | \, C_t^{Sp} \right), \quad T_0 \le t \le T_2 \tag{2}$$

3 Risk measure

The risk of losses of a position or a portfolio is assessed by means of risk measures. Risk measures ρ are real-valued mappings defined on linear spaces of random variables. In [1] desirable properties of risk measures were suggested from an economic point of view, in particular, their convexity and coherence. An important example of a one-period coherent risk measure is the Conditional-Value-at-Risk [9] at level $\alpha \in (0, 1)$ given by

$$\rho(z) := CVaR_\alpha(z) := \min_{r \in \mathbb{R}} \left\{ r + \frac{1}{\alpha} \mathbb{E} \left[\max\{0, -r - z\} \right] \right\}. \tag{3}$$

Here, one-period refers to z being a real-valued random variable. A significant advantage of $CVaR$ consists in its polyhedrality if z has finitely many scenarios. Hence, it may be incorporated into optimization models by introducing some additional variables and linear constraints. The latter property is shared by the multiperiod risk measures studied in [3]. There, several multiperiod extensions of $CVaR$ are suggested, that measure the risk of random vectors $(z_1, \dots, z_T)$. An example is the coherent risk measure

$$\rho(z_1, \dots, z_T) := \frac{1}{T-1} \inf \left\{ y_1^{(2)} + \sum_{t=2}^{T} \frac{1}{\alpha} \mathbb{E}\left[y_t^{(1)} \right] \middle| \begin{array}{l} y_1 \in \mathbb{R} \times \mathbb{R}, \; y_T \in \mathbb{R}_+ \times \mathbb{R}_+, y_t \in \mathbb{R}_+ \times \mathbb{R} \\ z_1 = -y_1^{(1)}, z_t = -y_t^{(1)} + y_t^{(2)} - y_{t-1}^{(2)}, \; t > 1 \end{array} \right\}. \tag{4}$$

We refer to [4] for further examples and some numerical experience with mean-risk electricity portfolio management models.

4 Optimization model

As the optimization problem is solvable only for a limited number of scenarios, it does not seem too restrictive to narrow the branching structure of the original $\mathcal{N}$-tree. We branch in the existing model once a month and reduce thereby future trading activities. A portfolio switching is made at the end of a month. Mathematically, we model this restricted structure by a second scenario tree with a node set $\mathcal{M} \subset \mathcal{N}$. The $\mathcal{M}$-tree comprises all nodes from the $\mathcal{N}$-tree coinciding with the last hour of every month and $m = 1$ corresponds to $n = 1$.

In view of medium-term optimization horizon, modelling of the thermal units of the power plant is reduced to upper and lower bounds for the power production and a maximum power variation velocity during one hour. So the model comprises only two integer variables among a large number of real variables. Hence, the problem is for a limited number of scenarios solvable with commercial solvers (CPLEX). Modelling of bid behavior in the auction trade system of EEX spot market is not an essential aim of the existing model. Hence, spot volumes are treated like balance energy with spot prices.

In the first years following the deregulation of energy markets, municipal power utilities normally placed long- or medium-term supply contracts with large power companies in addition to customer generation. The form of supply contract considerably differs with respect to flexibility and charges. Therefore, we modelled two types of contracts: a fixed and a flexible supply contract. The former is placed with a term of one year for a fixed price. Delivery will be made as agreed, subsequent changes of supply quantities are not possible.

The form of the flexible supply contract allows for an adjustment of agreed quantities within certain bounds during specified time periods. Equations (5) and (5) reflect the readjustment in node n of monthly scheduled quantities $P_n^{\mathrm{O,M}}$ based on the fixed yearly supply schedule $P_{t(n)}^{\mathrm{V}}$ at time $t(n)$ and the correction of daily supply schedules $P_n^{\mathrm{O,T}}$ based on the monthly quantities.

$$(1-\alpha) \cdot u^{\mathrm{O}} \cdot P_{t(n)}^{\mathrm{V}} \le P_n^{\mathrm{O,M}} \le (1+\alpha) \cdot u^{\mathrm{O}} \cdot P_{t(n)}^{\mathrm{V}}, \qquad (1-\beta) \cdot P_n^{\mathrm{O,M}} \le P_n^{\mathrm{O,T}} \le (1+\beta) \cdot P_n^{\mathrm{O,M}} \tag{5}$$

Here u^{O} denotes the binary decision variable for the flexible supply contract. Note that for the final daily quantities holds $P_n^{\mathrm{O,T}} = 0$ if $u^{\mathrm{O}} = 0$. The declaration of monthly supply quantities takes place in the previous month, i.e., the information structure of the associated decision variable is not in accordance with the data scenario tree. Therefore, we add explicit non-anticipativity restrictions for monthly supply quantities to the model. For the flexible supply contract the customer has to pay an energy rate depending on time and on actual power volumes and a demand rate depending on maximum annual power.

All these physical components of the portfolio are coupled by the load constraint, where P_n^{El} denotes the power production and P_n^{Sp} volumes traded on the spot market at node n as well as $P_{t(n)}^{\mathrm{RE}}$ a fixed portion of renewal energies at time $t(n)$.

$$L_n = P_n^{\mathrm{El}} + P_n^{\mathrm{O,T}} + u^{\mathrm{V}} P_{t(n)}^{\mathrm{V}} + P_n^{\mathrm{Sp}} + P_{t(n)}^{\mathrm{RE}} \tag{6}$$

Within the optimization model we allow for free future trading without any artificial time restrictions. In interaction with the risk measure a moderate future trading may be expected, hence, we can understand this financial portfolio component as a hedging instrument. At the European Energy Exchange (EEX) futures for various delivery periods, month, quarter and year, are traded. With regard to the optimization horizon of one year we model the former two and in each case the load types base and peak. Tradable are the next six months and the current delivery month and the respectively next seven quarters. Quarter contracts are fulfilled by cascading. Cascading means automatically splitting into three month contracts of the respective quarter contract on the last trading day before the transition to the delivery period takes place. Month contracts are fulfilled by cash settlement. Futures are characterized by a daily profit and loss equation and by the obligation to deposit securities.

For the sake of clarity, we by way of example describe the future balance for a month future taking into account a cascading quarter future of the same load type. The variables $F_{m,k}^{M,cas}$, $F_{m,l}^{M,cas}$ and $F_{m,r}^{M,cas}$ are auxiliary variables. Let $t(k) = t(m) + 1$, $t(l) = t(k) + 1$, $t(r) = t(l) + 1$ correspond to three consecutive months and H_k, H_l, H_r denote the number of delivery hours of the respective month futures. In equation (7) the number $F_{m,k}^{M}$ of a month future with delivery period k in a node m is updated. The balance contains the number of futures in previous month $F_{m-,k}^{\mathrm{M}}$, the number of purchased futures $F_{m,k}^{\mathrm{M,p}}$ and the number of sold futures $F_{m,k}^{\mathrm{M,s}}$ in node m as well as month futures $F_{m,k}^{M,cas}$ stemming from a cascading quarter future, where $F_{m-,m}^{Q}$ denotes the remaining number of quarter futures cascading at time $t(m)$.

$$\begin{aligned} F_{m,k}^{\mathrm{M}} &= F_{m-,k}^{\mathrm{M}} + F_{m,k}^{\mathrm{M,p}} - F_{m,k}^{\mathrm{M,s}} + F_{m,k}^{\mathrm{M,cas}} && (7)\\ F_{m,k}^{\mathrm{M,cas}} &= F_{m-,m}^{\mathrm{Q}} \cdot H_k \,/\, (H_k + H_l + H_r), \quad F_{m-,m}^{\mathrm{Q}} = F_{m--,m}^{\mathrm{Q}} + F_{m-,m}^{\mathrm{Q,p}} - F_{m-,m}^{\mathrm{Q,s}} && (8)\end{aligned}$$

These balance equations are formulated for all involved future types, month and quarter, and the load types base and peak. Further we added lower and upper bounds for the future stock, constraints for trading periods and time constraints related to cascading quarter futures to the model. When opening a position, a basic security, the so-called initial margin, must be deposited. The initial margin is bound for the entire duration of the contract. The change in value of the futures position results from the difference between the settlement price of the current day and the settlement price of the previous day. This change in value multiplied with the number of contracts leads to daily credit notes or additional payments which are called variation margin. Neglecting transaction fees we can assign the following financial transactions to the month future considered in (7).

$$\begin{aligned} z_m^{\mathrm{f,M}} = {} & z_{m-}^{\mathrm{f,M}} + C^{\mathrm{f,IM,M}}(F_{m,k}^{\mathrm{M,s}} - F_{m,k}^{\mathrm{M,p}}) + F_{m,k}^{\mathrm{M,cas}}(C^{\mathrm{f,IM,Q}} - C^{\mathrm{f,IM,M}}) + \\ & F_{m-,k}^{\mathrm{M}}(C_{m,k}^{\mathrm{f,M}} - C_{m-,k}^{\mathrm{f,M}}) + F_{m,k}^{\mathrm{M,cas}}(C_{m,k}^{\mathrm{f,M}} - C_{m-,m}^{\mathrm{f,Q}}) + F_{m-,m}^{\mathrm{M}} C^{\mathrm{f,IM,M}} \end{aligned} \tag{9}$$

The future cash flow in (9) is composed of the previous month's cash value, payment and repayment of initial margins, correction of initial margins due to cascading, variation margin of month future, variation margin of cascaded futures and repayment of initial margin for expired futures. Let now z_m^{f} denote the sum of all $z_m^{\mathrm{f,M}}$ and $z_m^{\mathrm{f,Q}}$ for all month and quarter futures, respectively, that enter in the balance sheet at node m. Further let z_m denote the book value of the portfolio at node m including the whole cash inflow and outflow. In particular, we have to consider total revenue from spot and futures market, payments from consumers of electricity, expenditures for power production, for supply contracts as well as for renewable energies. The consumer price C^{G} is a fixed artificial parameter. In addition to periodical payments, a demand rate depending on maximum annual power must be paid in the leafs of the tree. Let $m \in \mathcal{M}$ with $1 < t(m) < T$ and $m-$ correspond to $l \in \mathcal{N}$ and $r \in \mathcal{N}$ respectively.

$$z_m = z_{m-} + z_m^{\mathrm{f}} + \sum_{\substack{n \in \mathrm{path}(l),\\ t(r) < t(n) \le t(l)}} \Big(L_n C^{\mathrm{G}} - C_n^{\mathrm{Sp}} P_n^{\mathrm{Sp}} - C_{t(n)}^{\mathrm{El}} P_n^{\mathrm{El}} - C_{t(n)}^{\mathrm{RE}} P_{t(n)}^{\mathrm{RE}} - u^V C_{t(n)}^{\mathrm{V}} P_{t(n)}^{\mathrm{V}} - C_{t(n)}^{\mathrm{O,T}} P_n^{\mathrm{O,T}} \Big) \tag{10}$$

In order to model a risk-oriented profit maximization, the objective function appears as a weighted sum of the mean portfolio book value at the end of the optimization horizon and of a risk measure ρ depending on the book value at finitely many time periods.

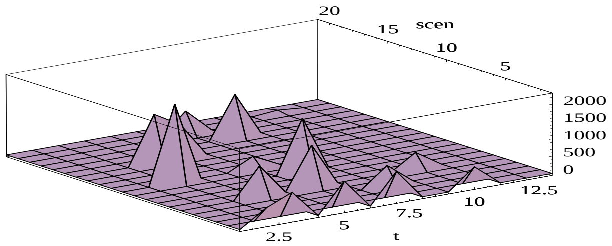
$$f(\gamma, z) = (1-\gamma) \textstyle\sum_{m \in \mathcal{M}_T} \pi_m z_m - \gamma \rho(z), \quad \gamma \in [0,1] \tag{11}$$

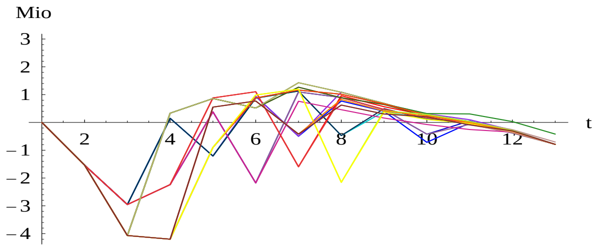
The weighting factor γ controls the relation between profit maximization and risk aversion. The book value of the portfolio in a node m is composed of all monthly debits and credits for the portfolio elements: power production, power supply contracts, EEX power contracts, supply due to legislation on the priority of renewable energies and futures.

5 Numerical results

In order to model and solve the optimization problem, we have used GAMS 21.1 and CPLEX 8.1 respectively. Numerical calculations were run for 21 scenarios leading to 2.4 millions of real variables. The experiments reported here are intended to investigate the composition of the portfolio and the distribution function of the book value for the CVaR and the multiperiod risk measure defined by (3) and (4), respectively.

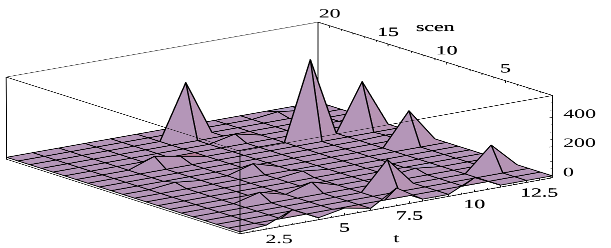
In particular, in case of CVaR the book value of the portfolio shows a high spread for the scenarios throughout the whole year except for the end of the time horizon. The portfolio consist of the same physical components independently of γ: the fixed supply contract, power produced by the own generation system and EEX spot market contracts. Further extensive future

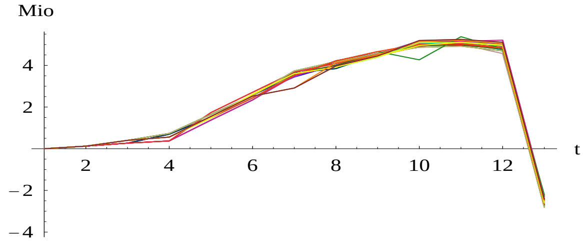




trading activities take place which are similar in size independently of γ. Computing times on a machine with 2 GHz Intel Celeron CPU and 512 MB RAM average out 3 hours.

Next we proceed to the multiperiod risk measure. This measure reduces the spread of the book value scenarios throughout the whole optimization horizon. For small γ the portfolio composition coincides with the former, whereas $\gamma = 0.5$ leads to a portfolio switching and to higher computing times of about 8 hours. The fixed supply contract is replaced by the more





expensive flexible supply contract. Apparently, due to the small number of branchings in the scenario tree, future trading activities are not able to control the book value taking into account the constraints for the multiperiod risk measure. However, the portfolio switching is associated with quantitatively increased future trading activities in contrast to the case $\gamma < 0.5$.

Further quantitative seasonal differences for the different physical portfolio components: volumes from the spot market, renewable energies, power production and the volumes of the flexible supply contract can be observed. During summer time power contracts at the spot market are predominantly sold. They are purchased only during daily peak load times. The flexible supply contract is in use the whole day. During winter time the situation is different: power contracts are purchased very often and the flexible supply contract is in use only during peak load times and almost never at weekends.

Acknowledgements This research was supported by the BMBF under grant 03RLM5B3. The authors would like to thank H. Heitsch (Humboldt University) for his assistance in providing the scenario tree as well as for useful hints and discussions.

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