PRIMAL AND DUAL METHODS FOR UNIT COMMITMENT IN A HYDRO-THERMAL POWER SYSTEM

R. Gollmer[1], A. Möller[2], M.P. Nowak[3], W. Römisch[3], R. Schultz[1]

[1] Gerhard-Mercator-Universität Duisburg, FB Mathematik, Lotharstr. 65, D-47048 Duisburg, Germany
[2] Weierstraß-Institut für Angewandte Analysis und Stochastik, Mohrenstr. 39, D-10117 Berlin, Germany
[3] Humboldt Universität Berlin, Institut für Mathematik, Unter den Linden 6, D-10099 Berlin, Germany

Abstract: For unit commitment in a real power generation system comprising thermal and pumped-storage hydro units a large-scale mixed-integer optimization model is developed and solved by primal and dual approaches. Solution methods use state-of-the-art algorithms and software. Results of test runs are reported.

Keywords: Unit commitment, mixed-integer linear programming, polyhedral combinatorics, Lagrangian relaxation, bundle methods

1 INTRODUCTION

Unit commitment in power operation planning aims at the cost optimal scheduling of on/off decisions and output levels for generating units. The power mix of the generation system has an essential impact on the design of mathematical models and algorithms for solving unit commitment problems. In the present paper, the interaction of a fair number of big coal fired blocks with several pumped storage plants of differing efficiencies provides the main challenge. This reflects the energy situation encountered at the German utility VEAG Vereinigte Energiewerke AG Berlin. Employing modern tools from mathematical optimization we demonstrate how to solve unit commitment problems for the VEAG system ranging over time horizons of up to 6 months with hourly discretizations. Solving here means that we are able to establish schedules whose objective function values provably are only per mills away from those of the optimal schedules. This makes an essential difference to local search heuristics ([1], [17]) such as tabu search, simulated annealing, and genetic algorithms that try to iteratively improve feasible schedules without being able to provide any certificates in the above sense. The paper starts with the mathematical model followed by primal and dual solution approaches both accompanied by reports on some characteristic test runs.

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2 MODEL

In our model, T is the number of subintervals of the optimization horizon, I, J are the numbers of thermal and pumped storage hydro units. The variable $\boldsymbol{u}_i^t \in \{0,1\},\ i = 1,\dots,I; t = 1,\dots,T$ indicates whether the thermal unit i is in operation at time t. Variables $\boldsymbol{p}_i^t, \boldsymbol{s}_j^t, \boldsymbol{w}_j^t, i = 1,\dots,I; j = 1,\dots,J;\ t = 1,\dots,T$ are the output levels for the thermal units, the hydro units in generation and in pumping modes, respectively. The variables $\boldsymbol{l}_j^t$ denote the fill (in energy) of the upper dam of the hydro unit j at the end of interval t, $j = 1,\dots,J;\ t = 1,\dots,T$.

The objective function to be minimized reads

$$\sum_{t=1}^{T}\sum_{i=1}^{I} C_i(\boldsymbol{p}_i^t, \boldsymbol{u}_i^t) + \sum_{t=1}^{T}\sum_{i=1}^{I} S_i^t(\boldsymbol{u}_i). \qquad (1)$$

Here, C_i denotes the fuel costs for unit i which often are a convex function of power output. We will consider linear and piecewise linear versions of C_i. The start-up costs $S_i^t(\boldsymbol{u}_i)$ of the i-th unit depend on its preceding down time.

When formulating the constraints our accent is on linear terms although elegant nonlinear alternatives exist. Sticking to linearity is motivated by the far more powerful mathemati-

cal tools available then (LP-based branch-and-bound and Lagrangian relaxation, polyhedral combinatorics).

Bounds for the power output of units and the fill of the upper dams read

$$\begin{array}{rl} p_{it}^{min}\boldsymbol{u}_i^t \le \boldsymbol{p}_i^t \le p_{it}^{max}\boldsymbol{u}_i^t, & i=1,\dots,I; \\ 0 \le \boldsymbol{s}_j^t \le s_{jt}^{max}, & j=1,\dots,J; \\ 0 \le \boldsymbol{w}_j^t \le w_{jt}^{max}, & t=1,\dots,T. \\ 0 \le \boldsymbol{l}_j^t \le l_j^{max}, & \end{array} \quad (2)$$

Here, $p_{it}^{min}, p_{it}^{max}, s_{jt}^{max}, w_{jt}^{max}$ denote minimal and maximal outputs, respectively, and l_j^{max} is the maximal fill of the upper dam.

The equilibrium between total generation and electrical load is covered by the equations

$$\sum_{i=1}^{I} \boldsymbol{p}_i^t + \sum_{j=1}^{J} (\boldsymbol{s}_j^t - \boldsymbol{w}_j^t) \ge D^t, \quad t=1,\dots,T, \quad (3)$$

where D^t denotes the electrical load at time t. Moreover, at each time, a spinning reserve R_t has to be ensured which is modeled by

$$\sum_{i=1}^{I} (\boldsymbol{u}_i^t p_{it}^{max} - \boldsymbol{p}_i^t) \ge R^t, \quad t=1,\dots,T. \quad (4)$$

For the whole time horizon, balances in the pumped storage plants have to be maintained:

$$\begin{array}{rlll} \boldsymbol{l}_j^t &= \boldsymbol{l}_j^{t-1} - (\boldsymbol{s}_j^t - \eta_j \boldsymbol{w}_j^t), & j{=}1,\dots,J; \\ \boldsymbol{l}_j^0 &= l_j^{in}, \qquad \boldsymbol{l}_j^T = l_j^{end}, & t{=}1,\dots,T. \end{array} \quad (5)$$

Here, l_j^{in}, l_j^{end} are the initial and final fills (in energy) of the upper dams, η_j denote the pumping efficiencies. At least in Germany, the VEAG system is somehow unique with its share of pumped storage plants that allows (and enforces) permanent utilization for economic purposes, in contrast to usage of pumped storage energy in peak periods and for emergencies only. Test runs for the VEAG system have shown that, for this economic purpose and considering the large-scale character of the decision problem, the linear approximation in (5) satisfactorily matches the nonlinear efficiency profile in practice.

Constraints avoiding simultaneous generation and pumping in the hydro plants are dispensable since such a deficiency can not occur in optimal points. In view of our time discretization into hourly intervals, the issue of ramping is not as critical as it would be with a finer discretization. This was confirmed by our test runs, and ramping, therefore does not occur in our one-hour-discretization model.

Finally, we have minimum down times τ_i for the thermal units. These are modeled via

$$\begin{array}{rll} \boldsymbol{u}_i^{t-1} - \boldsymbol{u}_i^t \le 1 - \boldsymbol{u}_i^l, & i=1,\dots,I; \\ & t=2,\dots,T{-}1; \\ & l=t{+}1,\dots,t{+}\tau_i \end{array} \quad (6)$$

where the constraints for the time intervals $t > T - \tau_i + 1$ have to be modified accordingly.

3 PRIMAL METHODS

LP-based branch-and-bound is among the earliest mathematical approaches to unit commitment, cf. [19]. It is based on formulating, possibly after exploiting proper equivalences, the unit commitment problem as a mixed-integer linear program that quickly becomes large-scale. While running branch-and-bound one has upper and lower bounds for the unknown optimal value. The relative difference between the least upper bound and the minimum lower bound provides a certificate of how close optimality has been reached. The problem being large-scale, a zero certificate is rather utopic, and certificates in the lower per cents or per mills are usually accepted as sufficiently good.

Early branch-and-bound approaches to unit commitment suffered from the comparatively poor mathematical methodology and software technology at that time. Meanwhile, this has changed drastically, both with respect to mathematical algorithms and software implementations, let alone hardware advances. General purpose codes like the CPLEX Callable Library [2] combine latest LP-methodology with a variety of options for arranging the branch-and-bound. In fact, the CPLEX Callable Library forms the algorithmic backbone of our primal approach to unit commitment.

To make LP-based branch-and-bound work for the above model the costs in (1) have to be expressed by (mixed-integer) linear terms. With the fuel costs C_i, this is possible for the (piecewise) linear situations. For the start-up costs S_i which depend exponentially on the preceding downtime we used approximations via step functions. The numbers of linearity regions for C_i and steps for S_i proved critical for the model size and hence for memory requirements and run

model dimensions	variant with groups of aggregated units and fixed start-up costs			variant with individual units and a 3-step function for start-up costs		
	1 week	1 month	6 months	1 week	1 month	6 months
integer variables	2112	8184	56472	5420	20832	130320
real variables	9781	37867	217608	15210	65442	383033
constraints	8053	31237	204576	22902	83364	594619
nonzeroes	31448	121877	760110	196803	749430	6363009

Table 1: Model dimensions for both model variants

CPU-time and accuracy	variant with groups of aggregated units and fixed start-up costs			variant with individual units and a 3-step function for start-up costs		
	1 week	1 month	6 months	1 week	1 month	6 months
CPU-time / min	0:58.9	7:40.9	234:02.9	7:44.3	161:32.9	out of
accuracy bound / %	0.086	0.073	0.133	0.391	0.389	memory

Table 2: Computing times on a HP 9000 (770/J180) and accuracy bounds of the primal method

times. Therefore, proper selections based on the concrete VEAG data were made here. The data situation was also exploited for guiding the subdivision (branching start-up variables according to the load profile), for improving model properties (introducing integer instead of Boolean variables for units with identical design) and for a fast heuristic to find a first feasible solution. Moreover, some first experiments with cutting planes from polyhedral combinatorics [18] were made to tighten the lower bounds.

Test runs with real-life data were performed on an HP 9000 (770/J180). Time horizons considered are 1 week, 1 month, and 6 months, with an hourly discretization. The generation system included 34 thermal and 7 hydro units. The generating costs were approximated by a linear function. Two approximations to the start-up costs were made, leading to two different model variants. In the first variant the start-up costs were constant. This enables the reformulation as a general mixed integer problem by aggregating the groups of technically identical units. In the second variant a step function with three steps per unit was used for the start-up costs. This prohibits aggregation of units and leads to much bigger models with an increase in computing time. The tighter accuracy bounds in the first variant result from a smaller gap between the LP-relaxation and the feasible set. The first table displays the problem dimensions. The solution process, which for branch-and-bound theoretically could be continued until a satisfactory accuracy bound is achieved, was finished, when the bound dropped below 1%. This always happened with the first feasible solution, found by a problem-specific rounding heuristic, which normally is followed by a branch-and-bound procedure for the full problem. Computing times and accuracy bounds are in Table 2.

In general, the primal approach via LP-based branch-and-bound allows ample model enrichment as long as this is expressible in mixed-integer linear terms. In particular, further internal coupling of the model caused by the introduction of additional constraints is not critical. This is exploited when extending the above model towards more sophisticated reserve policies involving hydro units [7] or towards staggered fuel prices [6]. On the other hand, always the full model has to be handled which may become prohibitive even if advanced methods are used for the LP-relaxations. This paves the way for decomposition which will be discussed next.

4 DUAL METHODS

Dual methods called Lagrangian relaxations have become very popular in unit commitment (cf. [19], [20]). Recently, three aspects made Lagrangian relaxation attractive and applicable to large-scale unit commitment problems: the algorithmic progress for solving the nondifferentiable Lagrangian dual, the usually small relative duality gap and the progress in fast Lagrangian heuristics for good primal feasible solutions. Early approaches for solving the dual problem were based on subgradient methods

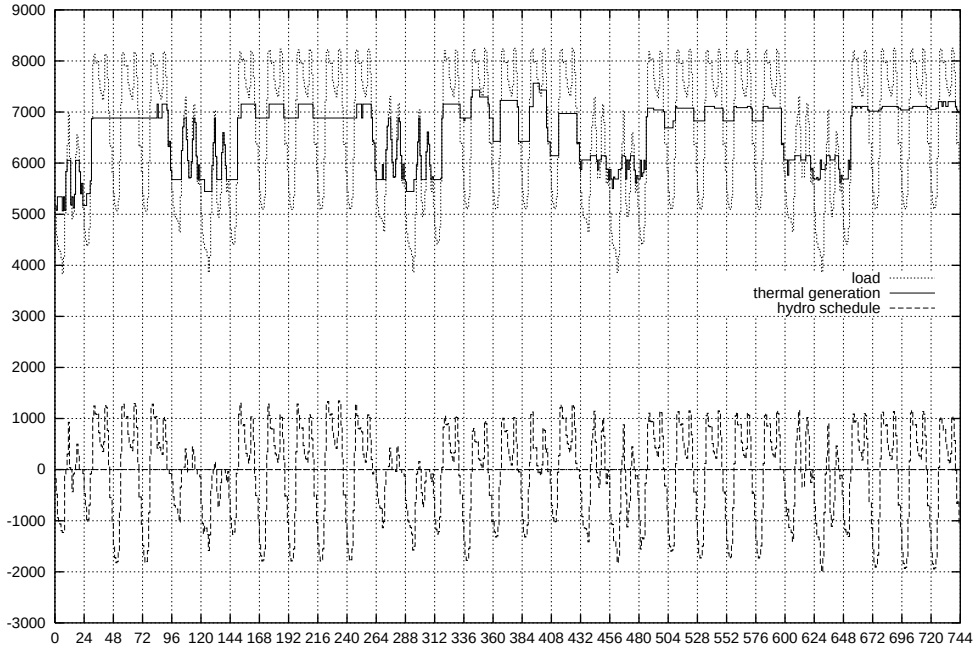

Figure 1: Solution of the primal method for 1 month

and smoothing techniques (cf. [19]). During the last decade more refined and efficient methods became popular: variants of cutting plane and bundle methods for convex nondifferentiable minimization (cf. [8]). We mention here dynamically constrained cutting plane methods ([9]), bundle-trust algorithms ([15]), reduced complexity bundle methods ([14]), variable metric bundle methods ([13]) and proximal bundle methods ([4], [5], [3], [7]). Moreover, dual convergence properties of proximal bundle methods are exploited in [4] to derive new Lagrangian heuristics for thermal systems and [12] provides a novel qualitative study of the duality gap for several Lagrangian relaxation schemes.

For the model in Section 2, our Lagrangian relaxation approach associates Lagrange multipliers with the loading constraints (3) and the modified reserve constraints

$$\sum_{i=1}^{I} \boldsymbol{u}_i^t p_{it}^{max} + \sum_{j=1}^{J} (\boldsymbol{s}_j^t - \boldsymbol{w}_j^t) \geq D^t + R^t, \qquad (7)$$

$(t = 1, \ldots, T)$. The dual problem reads

$$\max_{(\boldsymbol{\lambda},\boldsymbol{\mu}) \in I\!R_+^T \times I\!R_+^T} d(\boldsymbol{\lambda}, \boldsymbol{\mu}), \qquad (8)$$

where $\boldsymbol{\lambda}, \boldsymbol{\mu}$ are the Lagrange multipliers. The function d is defined by the infimum of the Lagrangian with respect to ($\mathbf{p}$, $\mathbf{u}$, $\mathbf{s}$, $\mathbf{w}$) under (2), (5), (6). d has the separable form

$$\begin{aligned} d(\boldsymbol{\lambda}, \boldsymbol{\mu}) := & \sum_{i=1}^{I} d_i(\boldsymbol{\lambda}, \boldsymbol{\mu}) + \sum_{j=1}^{J} \tilde{d}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ & + \sum_{t=1}^{T} [\boldsymbol{\lambda}^t D^t + \boldsymbol{\mu}^t (D^t + R^t)] \end{aligned} \qquad (9)$$

where the functions d_j, $\tilde{d}_j$ are optimal values of single-unit thermal and hydro subproblems:

$$\begin{aligned} d_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\boldsymbol{u}_i} \Big\{ & \sum_{t=1}^{T} [S_i(\boldsymbol{u}_i(t)) - \boldsymbol{\mu}^t \boldsymbol{u}_i^t p_{it}^{max} \\ & + \min_{\boldsymbol{p}_i^t} \{ C_i(\boldsymbol{p}_i^t, \boldsymbol{u}_i^t) - \boldsymbol{\lambda}^t \boldsymbol{p}_i^t \}] : \\ & \boldsymbol{u}_i^t, \boldsymbol{p}_i^t \text{ satisfy (2) and (6)} \Big\} \end{aligned}$$

$$\begin{aligned} \tilde{d}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\boldsymbol{s}_j, \boldsymbol{w}_j} \Big\{ & \sum_{t=1}^{T} (\boldsymbol{\lambda}^t + \boldsymbol{\mu}^t)(\boldsymbol{w}_j^t - \boldsymbol{s}_j^t) : \\ & \boldsymbol{s}_j^t, \boldsymbol{w}_j^t \text{ satisfy (2) and (5)} \Big\} \end{aligned}$$

The inner minimization of the thermal subproblems w.r.t. $\boldsymbol{p}_i^t$ is done explicitly while the outer minimization w.r.t. $\boldsymbol{u}_i$ is done by dynamic programming. For the hydro subproblems a fast descent algorithm from [16] is used. Since for the concave dual function d subgradients are available, powerful bundle-type algorithms [10] may be used for solving the Lagrangian dual (8). The optimal value of the dual provides a lower bound

for the minimal costs of the model in Section 2 and with the optimal multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ we have solutions of the thermal and hydro subproblems. In general these solutions cause violation of the load and reserve constraints (3) and (4) such that a low-cost (primal) feasible solution has to be determined by a Lagrangian heuristics. Altogether, the Lagrangian relaxation algorithm consists of the following steps:

Step 1: Initialize the multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$.

Step 2: Solve the dual problem (8) by the proximal bundle method ([10], [11]).

Step 3: Determine a primal feasible solution by a Lagrangian heuristics.

In both Step 2 and Step 3 thermal and hydro subproblems are solved repeatedly.

The multiplier $\boldsymbol{\lambda}$ is initialized by a list of thermal units in ascending order of relative costs at maximum output. In each time interval thermal units are switched on in list order until the total maximum output at least equals the demand or all units are online. The relative costs of the most expensive online unit initialize $\boldsymbol{\lambda}^t$. Initially, the multipliers $\boldsymbol{\mu}^t$ are zero in all intervals.

The proximal bundle method generates a sequence $(\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k)$ converging to some optimal multiplier and trial points $(\bar{\boldsymbol{\lambda}}_k, \bar{\boldsymbol{\mu}}_k)$ starting with $(\bar{\boldsymbol{\lambda}}_1, \bar{\boldsymbol{\mu}}_1) = (\boldsymbol{\lambda}_1, \boldsymbol{\mu}_1)$. The trial points are used for evaluating subgradients $g(\bar{\boldsymbol{\lambda}}_k, \bar{\boldsymbol{\mu}}_k)$ of the dual function d and its polyhedral upper approximation $\tilde{d}_k(\boldsymbol{\lambda}, \boldsymbol{\mu})$ defined by

$$\min_{j \in J_k} \{ d(\bar{\boldsymbol{\lambda}}_j, \bar{\boldsymbol{\mu}}_j) + g(\bar{\boldsymbol{\lambda}}_j, \bar{\boldsymbol{\mu}}_j)^T (\boldsymbol{\lambda} - \bar{\boldsymbol{\lambda}}_j, \boldsymbol{\mu} - \bar{\boldsymbol{\mu}}_j) \}$$

where J_k is a subset of $\{1, ..., k\}$. In iteration k the next trial point $(\bar{\boldsymbol{\lambda}}_{k+1}, \bar{\boldsymbol{\mu}}_{k+1})$ is selected to belong to

$$\operatorname{argmax} \{ \tilde{d}_k(\boldsymbol{\lambda}, \boldsymbol{\mu}) - \tfrac{1}{2} \sigma_k \| (\boldsymbol{\lambda} - \boldsymbol{\lambda}_k, \boldsymbol{\mu} - \boldsymbol{\mu}_k) \|^2 \}$$

where the maximization is subject to $(\boldsymbol{\lambda}, \boldsymbol{\mu}) \in \mathbb{R}_+^T \times \mathbb{R}_+^T$ and σ_k is a proximity weight. An ascent step to $(\boldsymbol{\lambda}_{k+1}, \boldsymbol{\mu}_{k+1}) = (\bar{\boldsymbol{\lambda}}_{k+1}, \bar{\boldsymbol{\mu}}_{k+1})$ occurs if $d(\bar{\boldsymbol{\lambda}}_{k+1}, \bar{\boldsymbol{\mu}}_{k+1}) \geq d(\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k) + \alpha v_k$, where $\alpha \in (0, 1)$ is fixed and $v_k = \tilde{d}_k(\bar{\boldsymbol{\lambda}}_{k+1}, \bar{\boldsymbol{\mu}}_{k+1}) - d(\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k)$. Otherwise a null step $(\boldsymbol{\lambda}_{k+1}, \boldsymbol{\mu}_{k+1}) = (\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k)$ improves the next polyhedral function $\tilde{d}_{k+1}$. General strategies for updating σ_k and choosing J_{k+1} are discussed in [10, 11]. The method is implemented in [11] such that the cardinality of J_k is bounded and that it terminates if v_k is less than a given (relative) optimality tolerance.

Two different Lagrangian heuristics were developed and implemented. The first heuristics (LH1) has three steps and starts with reducing the value $D^t + R^t + \sum_{j=1}^J [\boldsymbol{w}_j^t - \boldsymbol{s}_j^t]$ by modifying the schedule of the hydro plants if the reserve constraint (7) is violated at time t and the value of this sum is the largest in a certain set of intervals. This may cause new violations of the reserve constraint in intervals where the above sum is small. In a second step the hydro variables are fixed, and following [21] we search for binary variables $\boldsymbol{u}_i^t$ fulfilling $\sum_{i=1}^I \boldsymbol{u}_i^t p_{it}^{max} \geq D^t + R^t + \sum_{j=1}^J [\boldsymbol{w}_j^t - \boldsymbol{s}_j^t]$. The main idea is to take the interval where this condition is violated most and to compute the increase of μ^t necessary to switch on just that many thermal units such that the condition holds. This is repeated until the reserve constraint (7) holds in all intervals. After having fixed the binary variables $\boldsymbol{u}_i^t$, the economic dispatch problem is solved by CPLEX [2].

The Lagrangian heuristics LH2 exploits the structure of the dual problem (8) and screens all solutions $(\mathbf{p}, \mathbf{u}, \mathbf{s}, \mathbf{w})$ corresponding to (nearly) optimal multipliers $(\boldsymbol{\lambda}, \boldsymbol{\mu})$. For convex models any feasible primal solution $(\mathbf{p}, \mathbf{u}, \mathbf{s}, \mathbf{w})$ corresponding to optimal multipliers, is also optimal. This fails in the mixed-integer situation. Instead we determine a set of primal solutions corresponding to slightly perturbed optimal multipliers. To this end we first screen the binary decisions in the dynamic programming solutions to the thermal subproblems. Test runs showed that only a few of these variables change. Fixing the remaining binary decisions drastically reduces dimension. Then a decreasing sequence of binary decisions $\mathbf{u}$ is constructed. In each step a period t is selected where the available reserve $\sum_{i=1}^I (\boldsymbol{u}_i^t p_{it}^{max} - \boldsymbol{p}_i^t) - R^t$ is large, and the multipliers are used to determine in which preceding and consecutive periods some unit can be switched off. For each element of the sequence an economic dispatch problem is solved by a modification of the descent method from [16]. The element with the least optimal value provides a reasonably good solution of the problem (1)-(6).

The results in Table 3, Table 4 and Figure 2 are based on the same data and hardware as for the primal method. Compared with Section

NOA 3.0 optimality tolerance: 10^{-4}		optimization horizon					
		1 week		1 month		6 months	
production costs	start-up costs	time/ min	bound of gap/ %	time/ min	bound of gap/ %	time/ min	bound of gap/ %
linear	constant	0:17	1.10	2:36	0.93	60:15	0.84
linear	time dependent	0:20	1.13	3:04	0.98	63:04	0.73
piecew. lin.	constant	0:28	1.09	5:33	0.86	110:23	0.79
piecew. lin.	time dependent	0:30	1.07	5:28	0.96	119:02	0.69

Table 3: CPU-time in minutes on HP 9000 (770/J180) and upper bound of the duality gap of the dual method (with LH1)

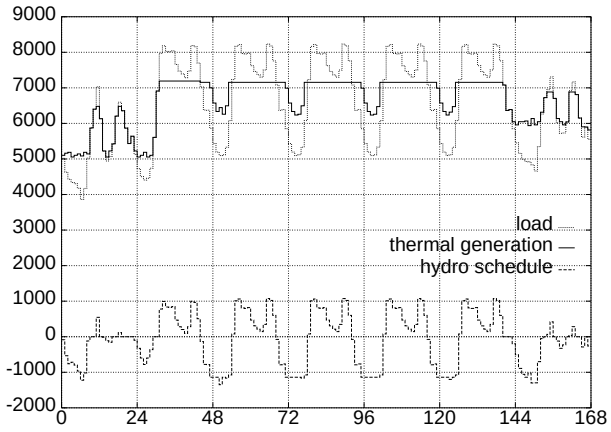

Figure 2: Solution of the dual method (with LH1) for 1 week

production costs	start-up costs	bound of gap/ %	time/ min
optimization horizon:		1 week	
linear	constant	0.44	0:19
linear	time dep.	0.28	0:18
piecew. lin.	constant	0.20	0:21
piecew. lin.	time dep.	0.29	0:22
optimization horizon:		1 month	
piecew. lin.	constant	0.30	6:24
piecew. lin.	time dep.	0.42	5:26

Table 4: Upper bound for the duality gap and CPU-time on HP 9000 (J280) of the dual method (with LH2)

3 exponential approximations for start-up costs leading to more than 50 different steps were used and examples with piecewise linear fuel costs were run. The dual method is faster than the primal but yields wider accuracy bounds. Refined modelling of down time dependence of start-up costs is less time critical in the dual approach since the time for solving the thermal subproblems is linear in the number of start-up cost approximation steps. The heuristics LH2 yields tighter accuracy bounds than LH1. Piecewise linear fuel costs cause quite substantial increases in computing time when using LH1 and CPLEX for the economic dispatch. This does not occur in LH2, where a specific descent algorithm for economic dispatch is employed.

5 CONCLUSIONS

The approaches to unit commitment presented in this paper are fast, powerful and supplement each other. The primal, branch-and-bound based method shows its strength with complex constraints interconnecting generation units. The dual algorithm, that relies on Lagrangian relaxation, has its merits with complicated, even nonlinear conditions imposed on single units but interconnected in time. Both approaches have advanced mathematics as an indispensable ingredient. In the primal case these are recent LP-methodology and a flexible and efficient branch-and-bound scheme. The dual method is an interplay of high-level non-smooth optimization, custom made algorithms for the single unit subproblems and Lagrangian heuristics, for which we presented two new proposals. To the best of our knowledge, the literature has no comparable contributions where a mid-size hydro-thermal system is provably optimized (with certificates in the per mills) over hourly discretized time horizons of up to 6 months.

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