Dynamic Risk Management
in Electricity Portfolio Optimization
via Polyhedral Risk Functionals

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Introduction

- Power optimization models often contain uncertain parameters, for which statistical data is available.
- The uncertain parameters may be represented approximatively by a finite number of scenarios and their probabilities.
- Scenarios become tree-structured if they appear in a process of recursive observations and decisions,
- Advantages of such stochastic programming models:
  - Decisions are robust with respect to random perturbations,
  - The risk of decisions can be modeled properly and minimized,
  - Simulation studies show the better performance.

Mathematical challenges

- Generation of scenarios from statistical models (e.g., simulation from time series models, resampling techniques),
- Generation of scenario trees out of given scenarios and their eventual reduction,
- Risk modeling and minimization,
- Decomposition methods of the resulting very large scale (mixed-integer) linear programming models (e.g., Lagrangian relaxation of coupling constraints).

Risk Functionals

A risk functional ρ assigns a real number to any (real) random variable Y (possibly satisfying certain moment conditions). Recently, it was suggested that ρ should satisfy the following axioms for all random variables Y, $\tilde{Y}$, $r \in \mathbb{R}$, $\lambda \in [0,1]$:

(A1) $\rho(Y + r) = \rho(Y) - r$ (translation-antivariance),

(A2) $\rho(\lambda Y + (1-\lambda)\tilde{Y}) \leq \lambda\rho(Y) + (1-\lambda)\rho(\tilde{Y})$ (convexity),

(A3) $Y \leq \tilde{Y}$ implies $\rho(Y) \geq \rho(\tilde{Y})$ (monotonicity).

A risk functional ρ is called coherent if it is, in addition, positively homogeneous, i.e., $\rho(\lambda Y) = \lambda\rho(Y)$ for all $\lambda \geq 0$ and random variables Y.
Given a risk functional ρ, the mapping $\mathcal{D} = \mathbb{E} + \rho$ is also called deviation risk functional.

References: Artzner-Delbaen-Eber-Heath 99, Föllmer-Schied 02, Fritelli-Rosazza Gianin 02

Examples:

(a) Average Value-at-Risk $\mathbb{A}\mathsf{V@R}_\alpha$:

$$\mathbb{A}\mathsf{V@R}_\alpha(Y) := \frac{1}{\alpha}\int_0^\alpha \mathbb{V}\mathsf{@R}_u(Y)(u)du$$
$$= \inf\left\{x + \frac{1}{\alpha}\mathbb{E}([Y+x]^-) : x \in \mathbb{R}\right\}$$
$$= \sup\left\{-\mathbb{E}(YZ) : \mathbb{E}(Z) = 1, 0 \le Z \le \frac{1}{\alpha}\right\}$$

where $\alpha \in (0,1]$, $\mathbb{V}\mathsf{@R}_\alpha := \inf\{y \in \mathbb{R} : \mathbb{P}(Y \le y) \ge \alpha\}$ is the Value-at-Risk and $[a]^- := -\min\{0, a\}$.

Reference: Rockafellar-Uryasev 02

(b) Lower semi standard deviation corrected expectation:

$$\rho(Y) := -\mathbb{E}(Y) + (\mathbb{E}([Y - \mathbb{E}(Y)]^-)^2)^{\frac{1}{2}}$$

Reference: Markowitz 52

Multi-Period Risk Functionals

Let $\xi = (\xi_1, \dots, \xi_T)$ be some input random vector. We assume that all random vectors $Y = (Y_1, \dots, Y_T)$ have the property that Y_t only depends on $(\xi_1, \dots, \xi_t)$.

A functional ρ that assigns to each such random vector $Y = (Y_1, \dots, Y_T)$ a real number is called a multi-period risk functional if it satisfies the following conditions for all random vectors $Y = (Y_1, \dots, Y_T)$ and $\tilde{Y} = (\tilde{Y}_1, \dots, \tilde{Y}_T)$:

(A1) $\rho(Y_1 + W_1, \dots, Y_T + W_T) = -\sum_{t=1}^{T} \mathbb{E}(W_t) + \rho(Y_1, \dots, Y_T)$ for all W belonging to some convex subset of random vectors $\mathcal{W}$ (possibly depending on ξ) ($\mathcal{W}$-translation-antivariance),

(A2) ρ is convex (convexity),

(A3) $Y_t \le \tilde{Y}_t$, for all t, implies $\rho(Y_1, \dots, Y_T) \ge \rho(\tilde{Y}_1, \dots, \tilde{Y}_T)$ (monotonicity).

The set $\mathcal{W}$ is related to the set of available financial instruments for hedging the risk.

References: Artzner-Delbaen-Eber-Heath-Ku 07, Fritelli-Scandolo 06, Pflug-Römisch 07

Example: (for the set $\mathcal{W}$)

(a) $\mathcal{W} = \{(x, 0, \ldots, 0) \in \mathbb{R}^T : x \in \mathbb{R}\} = \mathbb{R} \times \{0\}^{T-1}$

(Artzner-Delbaen-Eber-Heath-Ku 07).

(b) $\mathcal{W} = \mathbb{R}^T$.

(c) $\mathcal{W} = \{W = (W_1, \ldots, W_T) : \sum_{t=1}^{T} W_t$ is deterministic$\}$.

(Fritelli-Scandolo 06).

(d) $\mathcal{W} = \{W = (W_1, \ldots, W_T) : W_t$ depends only on $(\xi_1, \ldots, \xi_{t-1})\}$

(Pflug-Ruszczynski 04).

Polyhedral risk functionals:

Multi-period risk functionals are called polyhedral if they preserve linearity structures (stability and decomposition properties) of stochastic programming models (when inserted into them) although such functionals are nonlinear by nature. They may be represented by (classical) linear stochastic programs.

Reference (for polyhedral risk functionals): Eichhorn-Römisch 05.

Examples:

(a) Expectation of accumulated incomes $\sum_{\tau=1}^{t} Y_\tau$ at risk measuring time steps t_j, $j = 1, \ldots, J$, with $t_J = T$:

$$\rho_0(Y_{t_1}, \ldots, Y_{t_J}) := \sum_{j=1}^{J} \mathbb{E}\left(\sum_{t=1}^{t_j} Y_t\right)$$

(b) Sum of Average Value-at-Risk's at risk measuring time steps:

$$\rho_1(Y_{t_1}, \ldots, Y_{t_J}) := \frac{1}{J} \sum_{j=1}^{J} \mathbb{A}\mathsf{V@R}_\alpha \left(\sum_{t=1}^{t_j} Y_t\right)$$

(c) Average Value-at-Risk of the average at risk measuring time steps:

$$\rho_4(Y_{t_1}, \ldots, Y_{t_J}) := \mathbb{A}\mathsf{V@R}_\alpha \left(\frac{1}{J} \sum_{j=1}^{J} \sum_{t=1}^{t_j} Y_t\right)$$

(d) Average Value-at-Risk of the minimum at risk measuring time steps:

$$\rho_6(Y_{t_1}, \ldots, Y_{t_J}) := \mathbb{A}\mathsf{V@R}_\alpha \left(\min_{j=1,\ldots,J} \sum_{t=1}^{t_j} Y_t\right)$$

All examples are polyhedral risk functionals and satisfy $\mathbb{R} \times \{0\}^{T-1}$-translation-antivariance.

Stochastic programming problem with risk objective:

$$\min_{x} \left\{ \rho(Y_1, \ldots, Y_T) \;\middle|\; \begin{array}{l} Y_t = \langle b_t(\xi_t), x_t \rangle, \\ x_t = x_t(\xi_1, \ldots, \xi_t) \in X_t, \\ \sum_{\tau=0}^{t-1} A_{t,\tau}(\xi_t) x_{t-\tau} = h_t(\xi_t) \\ (t = 1, \ldots, T) \end{array} \right\}$$

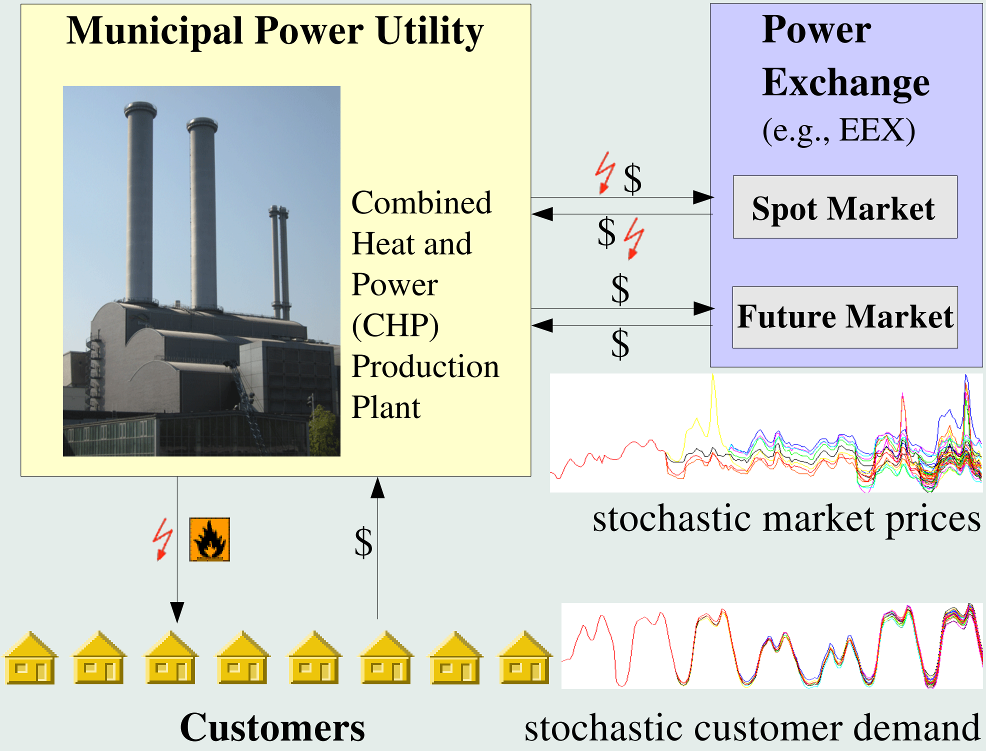
Polyhedral risk functional (evaluated at risk measuring time steps):

$$\rho(Y) = \inf \left\{ \mathbb{E}\left(\sum_{j=0}^{J} \langle c_j, v_j \rangle \right) \;\middle|\; \begin{array}{l} v_j = v_j(\xi_1, \ldots, \xi_{t_j}) \in V_j, \\ \sum_{k=0}^{j} B_{j,k} v_{j-k} = r_j \\ (j = 0, \ldots, J), \\ \sum_{k=0}^{j} \langle a_{j,k}, v_{j-k} \rangle = \sum_{t=1}^{t_j} Y_t \\ (j = 1, \ldots, J) \end{array} \right\}$$

Equivalent linear stochastic programming model:

$$\min_{(v,x)} \left\{ \mathbb{E}\left(\sum_{j=0}^{J} \langle c_j, v_j \rangle \right) \;\middle|\; \begin{array}{l} x_t = x_t(\xi_1, \ldots, \xi_t) \in X_t, \\ v_j = v_j(\xi_1, \ldots, \xi_{t_j}) \in V_j, \\ \sum_{s=0}^{t-1} A_{t,s}(\xi_t) x_{t-s} = h_t(\xi_t), \\ \sum_{k=0}^{j} B_{j,k} v_{j-k} = r_j, \\ \sum_{k=0}^{j} \langle a_{j,k}, v_{j-k} \rangle = \sum_{t=1}^{t_j} \langle b_t(\xi_t), x_t \rangle \end{array} \right\}$$

Mean-Risk Electricity Portfolio Management



We consider the electricity portfolio management of a German municipal electric power company. Its portfolio consists of the following positions:

- power production (based on company-owned thermal units),
- bilateral contracts,
- (physical) (day-ahead) spot market trading (e.g., European Energy Exchange (EEX)) and
- (financial) trading of futures.

The time horizon is discretized into hourly intervals. The underlying stochasticity consists in a multivariate stochastic load and price process that is approximately represented by a finite number of scenarios. The objective is to maximize the total expected revenue and to minimize the risk. The portfolio management model is a large scale (mixed-integer) multi-stage stochastic program.

Electricity portfolio management: statistical models and scenario trees

For the stochastic input data of the optimization model (here yearly electricity and heat demand, and electricity spot prices), a statistical model is employed. It is adapted to historical data in the following way:

- cluster classification for the intra-day (demand and price) profiles,

- 3-dimensional time series model for the daily average values (deterministic trend functions, a trivariate ARMA model for the (stationary) residual time series),

- simulation of an arbitrary number of three dimensional sample paths (scenarios) by sampling the white noise processes for the ARMA model and by adding on the trend functions and matched intra-day profiles from the clusters afterwards,

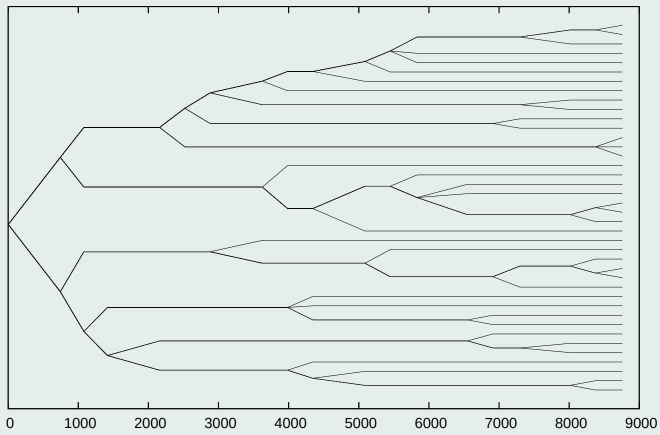
- generation of scenario trees (Heitsch-Römisch 05).

Electricity portfolio management: Results

Test runs were performed on real-life data of a German municipal power company leading to a linear program containing $T = 365 \cdot 24 = 8760$ time steps, a scenario tree with 40 demand-price scenarios (see below) with about 150.000 nodes. The objective function is of the form

$$\text{Minimize} \quad \gamma\rho(Y) - (1-\gamma)\mathbb{E}\left(\sum_{t=1}^{T} Y_t\right)$$

with a (multiperiod) risk functional ρ with risk aversion parameter $\gamma \in [0,1]$ ($\gamma = 0$ corresponds to no risk).



Single-period and multi-period risk functionals are computed for the accumulated income at $t = T$ and at the risk time steps t_j, $j = 1, \ldots, J = 52$, respectively. The latter correspond to 11 pm at the last trading day of each week.

It turns out that the numerical results for the expected maximal revenue and minimal risk

$$\mathbb{E}\left(\sum_{t=1}^{T} Y_t^{\gamma *}\right) \qquad \text{and} \qquad \rho(Y_{t_1}^{\gamma *}, \ldots, Y_{t_J}^{\gamma *})$$

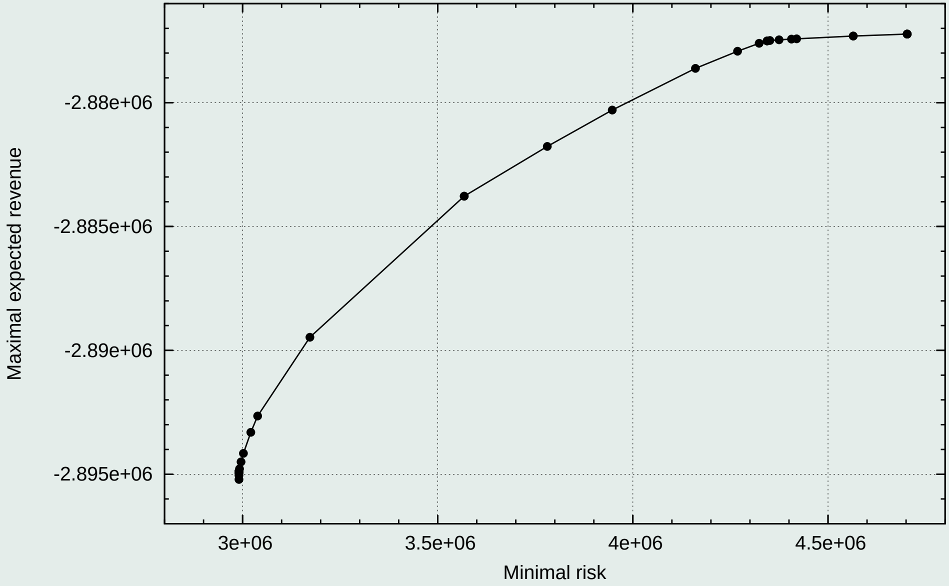
with the optimal income process $Y^{\gamma *}$ are **identical** for $\gamma \in [0.15, 0.95]$ and all risk functionals used in the test runs.

The efficient frontier

$$\gamma \mapsto \left(\rho(Y_{t_1}^{\gamma *}, \ldots, Y_{t_J}^{\gamma *}), \mathbb{E}\left(\sum_{t=1}^{T} Y_t^{\gamma *}\right)\right)$$

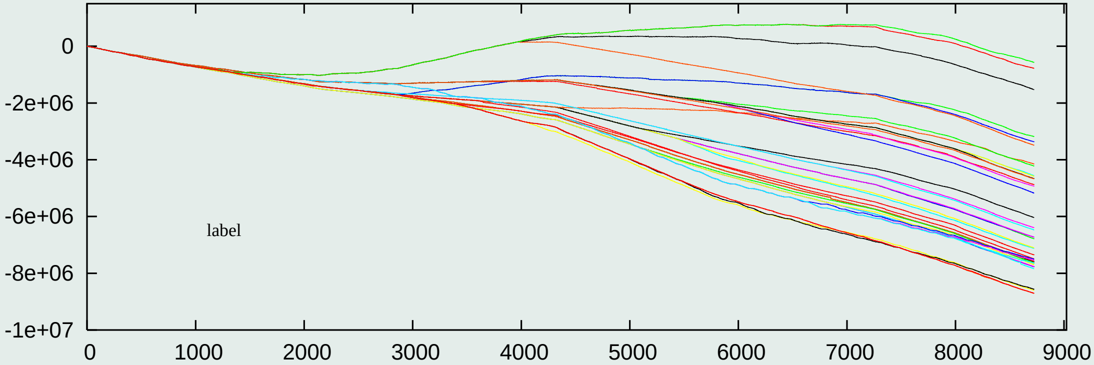
is concave for $\gamma \in [0, 1]$.

Risk aversion costs less than 1% of the expected overall revenue.

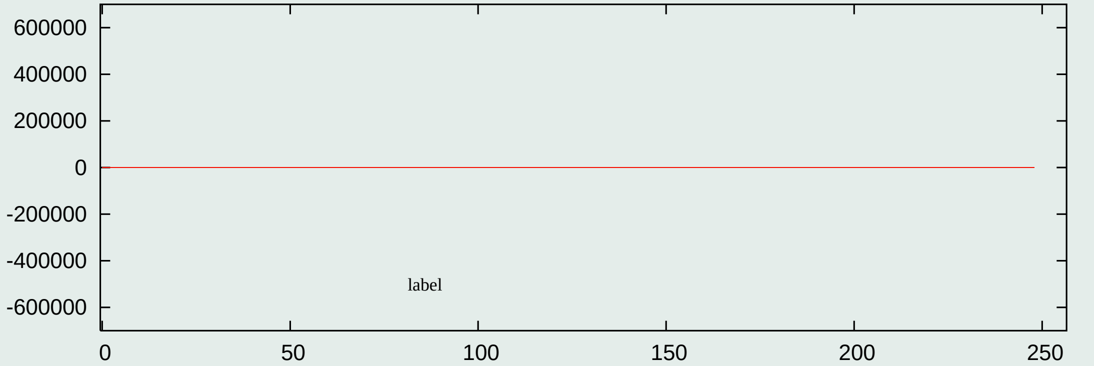


γ	$\rho_6(Y^{\gamma *})$	$\mathbb{E}\left(\sum_{t=1}^{T} Y_t^{\gamma *}\right)$
5.000000000000e-04	4.702811848065e+06	-2.877230530260e+06
1.000000000000e-03	4.374541317978e+06	-2.877465216029e+06
2.000000000000e-03	4.351171030788e+06	-2.877491632750e+06
3.000000000000e-03	4.344098951661e+06	-2.877507683071e+06
4.000000000000e-03	4.344098951661e+06	-2.877507683071e+06
5.000000000000e-03	4.323657436401e+06	-2.877601246036e+06
6.000000000000e-03	4.268097050724e+06	-2.877925822310e+06
7.000000000000e-03	4.160262308909e+06	-2.878617331737e+06
8.000000000000e-03	3.947240133128e+06	-2.880298275881e+06
9.000000000000e-03	3.780867553726e+06	-2.881767795158e+06
1.000000000000e-02	3.567995960437e+06	-2.883774406315e+06
2.000000000000e-02	3.172449940047e+06	-2.889472010420e+06
3.000000000000e-02	3.038501183774e+06	-2.892650974281e+06
4.000000000000e-02	3.020970182403e+06	-2.893311061998e+06
5.000000000000e-02	3.001973006480e+06	-2.894160037393e+06
6.000000000000e-02	2.996151299394e+06	-2.894502042746e+06
7.000000000000e-02	2.992169811757e+06	-2.894783498866e+06
1.000000000000e-01	2.990833381773e+06	-2.894890967278e+06
1.500000000000e-01	2.990664257637e+06	-2.894919429755e+06
2.000000000000e-01	2.990664257637e+06	-2.894919429755e+06
⋮	⋮	⋮
9.000000000000e-01	2.990664257637e+06	-2.894919429755e+06
9.500000000000e-01	2.990664257637e+06	-2.894919429755e+06
9.900000000000e-01	2.990664257637e+06	-2.894919431317e+06
9.990000000000e-01	2.990664257637e+06	-2.895039700908e+06
9.999000000000e-01	2.990664257637e+06	-2.895212797595e+06

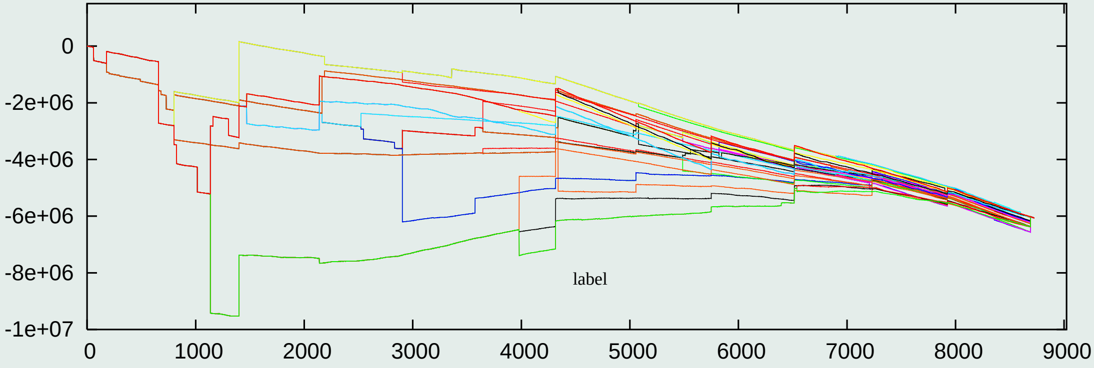
The LP is solved by CPLEX 9.1 in about 1 h running time on a 2 GHz Linux PC with 1 GB RAM.



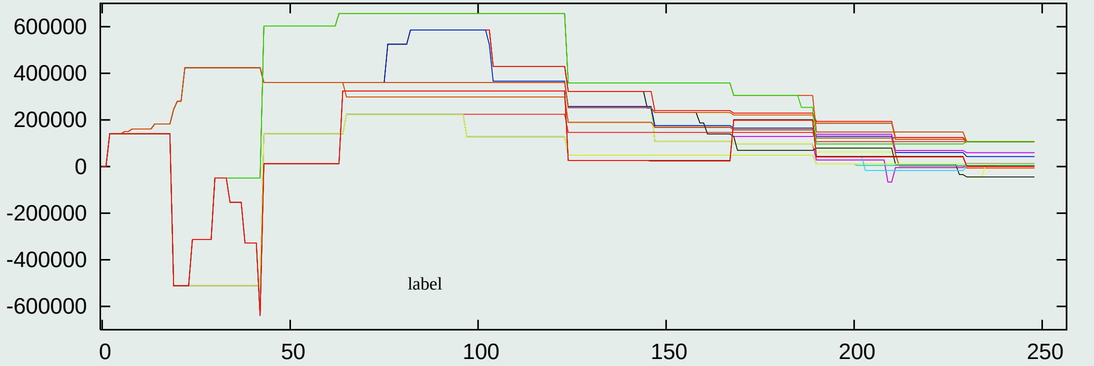
Overall revenue scenarios for $\gamma = 0$



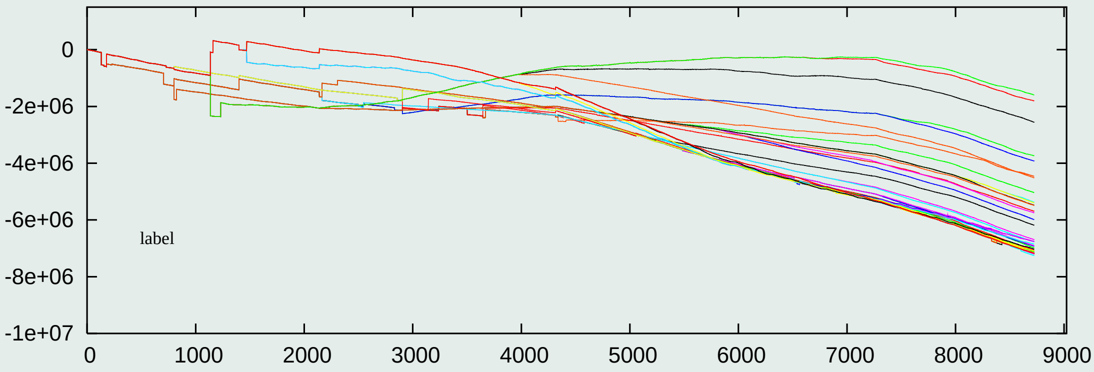
Future trading for $\gamma = 0.9$



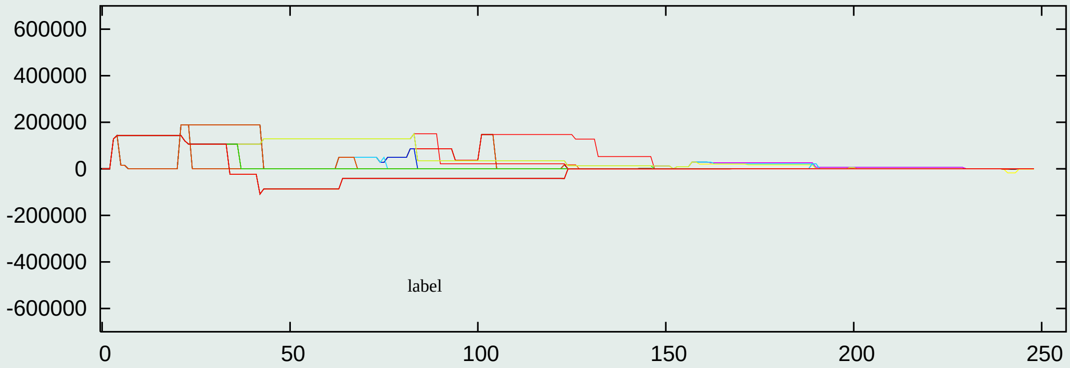
Overall revenue scenarios with $\mathbb{AV@R}_{0.05}$ and $\gamma = 0.9$



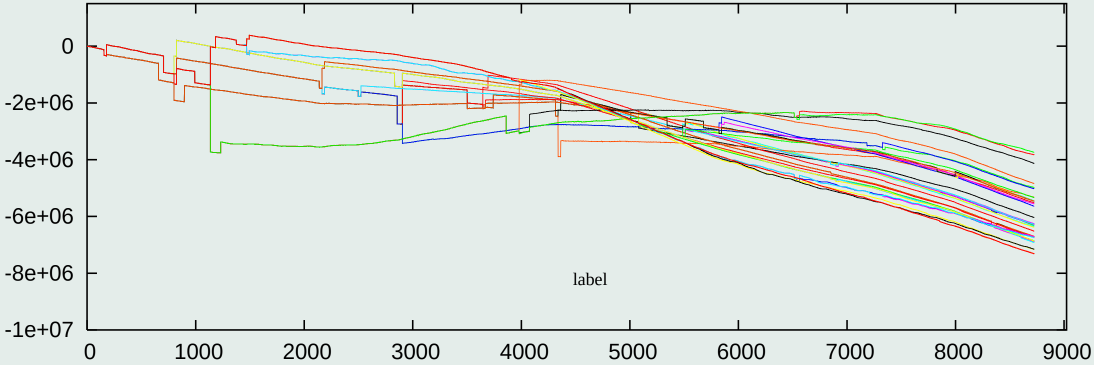
Future trading with $\mathbb{AV@R}_{0.05}$ and $\gamma = 0.9$



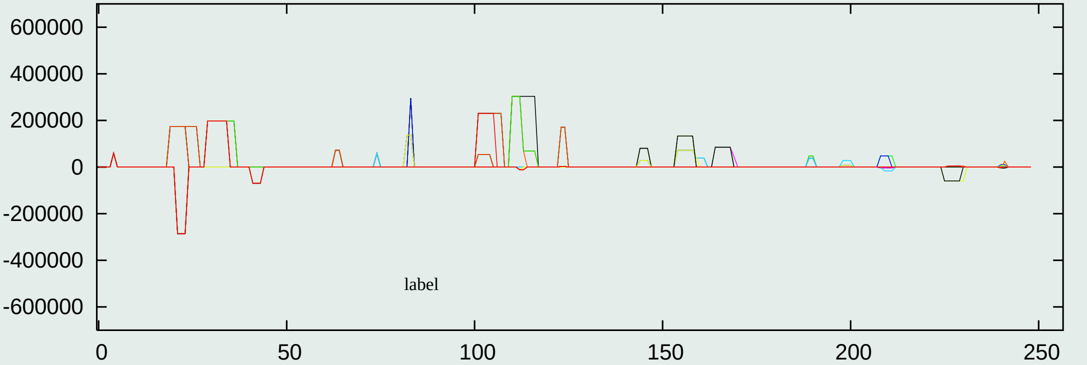
Overall revenue scenarios with ρ_1 and $\gamma = 0.9$



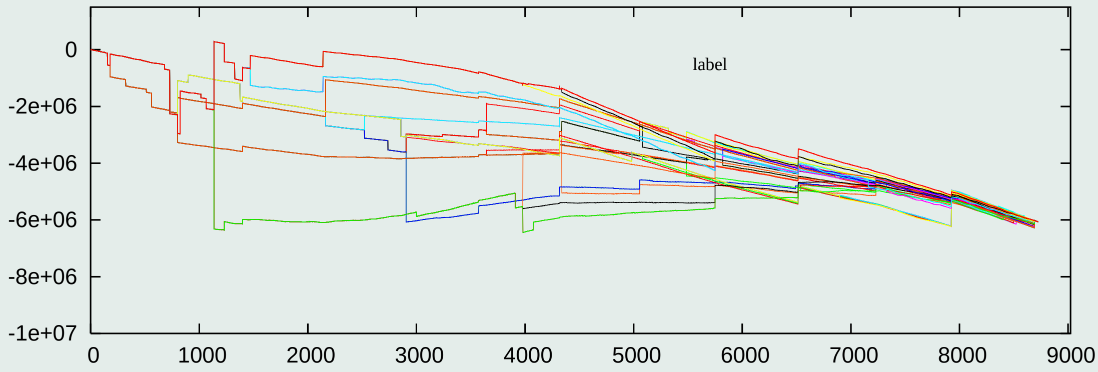
Future trading for ρ_1 and $\gamma = 0.9$



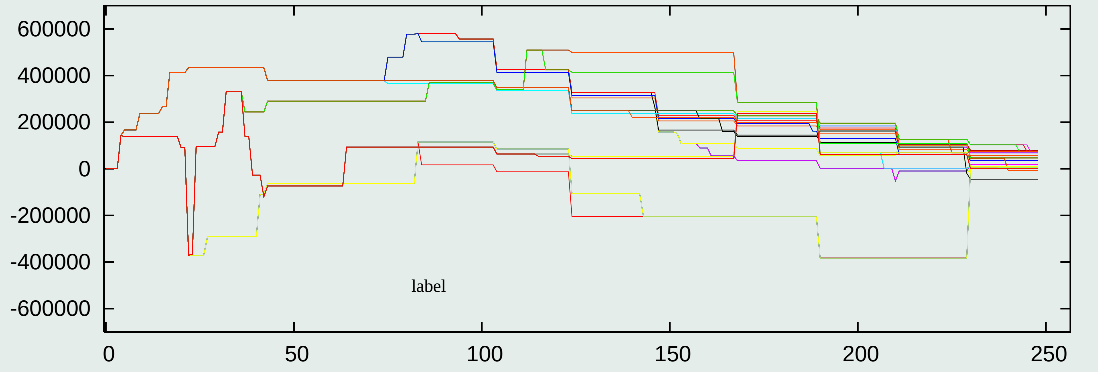
Overall revenue scenarios with ρ_4 and $\gamma = 0.9$



Future trading with ρ_4 and $\gamma = 0.9$



Overall revenue scenarios with ρ_6 and $\gamma = 0.9$



Future trading with ρ_6 and $\gamma = 0.9$

Thank you

References:

Eichhorn, A., and Römisch, W.: Polyhedral risk measures in stochastic programming, *SIAM Journal on Optimization* 16 (2005), 69–95.

Pflug, G. Ch.; Römisch, W.: *Modeling, Measuring and Managing Risk*, World Scientific, Singapore, 2007.