Humboldt-Universität zu Berlin
Institut für Mathematik

Advanced Topics in Optimization
Mathematical Image Processing
Summer semester 2018/19

Exercise Sheet 5

Consider the regularized version of the predual TV denoising problem:

(1) $$\min_{p \in H_0^1(\Omega)^2} \frac{\beta}{2}\|\nabla p\|_{L^2}^2 + \frac{\gamma}{2}\|p\|_{L^2}^2 + \frac{1}{2}\|\text{div}p + f\|_{L^2}^2 + \frac{1}{\epsilon}\mathcal{P}(p,\alpha)$$

where the term

$$\frac{1}{\epsilon}\mathcal{P}(p,\alpha) := \frac{1}{\epsilon}\int_\Omega \sum_{i=1}^{2}(G(p_i-\alpha)+G(-(p_i+\alpha)))$$

aims to approximately enforce the constraint $-\alpha \le p_i \le \alpha$, $i = 1, 2$, $p = (p_1, p_2)$. Here $G : \mathbb{R} \to \mathbb{R}$ with

$$G(r) := \begin{cases} \frac{1}{2}r^2, & \text{if } r > 0, \\ 0, & \text{if } r < 0, \end{cases}$$

Use the Semismooth Newton Method to solve the first order optimality condition of (1):

(2) $$F(p) := -\beta\Delta p + \gamma p - \nabla\text{div}p - f + \frac{1}{\epsilon}P(p,\alpha) = 0$$

with P being the derivative of $\mathcal{P}$ (with respect to p).

$$P(p,\alpha) := \max(p-\alpha, 0) - \max(-p-\alpha, 0)$$

Instructions:

- The noisy data f are on the course website.
- Define your mesh size h (e.g. $D_x u = \frac{u_{i+1,j}-u_{i,j}}{h}$) for your differential operators as $h = 1/\sqrt{nm}$ where $n \times m$ is the image resolution.
- The Laplacian and the ∇div operator must be constructed with the correct boundary conditions (zero Dirichlet).
- The parameters β, γ, ϵ have to be relatively small in order to approximate the original predual TV problem.
- Experiment with different values of the regularization parameter α and observe the differences in the reconstruction result.