18.01 SPRING 2005
MIDTERM 2 PREPARATION

Material covered

Midterm 2 will be Friday, April 8, and will cover the material from Problem Sets 4 through 7. Material from Problem Set 8 (advanced partial fractions and integration by parts) will not be included; there may be *simple* partial fractions problems, along the same lines as Part B #1a from Problem Set 7.

Formula sheet

The format of the exam will be the same as for Midterm 1, except that you will also be given a short formula sheet, including the following information:

1. Some useful antiderivatives:

$$\int \sec^2 x \; dx = \tan x + C \qquad \int \csc^2 x \; dx = -\cot x + C$$

$$\int \sec x \; dx = \ln|\sec x + \tan x| + C \qquad \int \csc x \; dx = \ln|\csc x - \cot x| + C$$

$$\int \sec x \tan x \; dx = \sec x + C \qquad \int \csc x \cot x \; dx = -\csc x + C$$

2. Triogonometry:
   - $\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$
   - $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}, \quad \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
   - $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

General advice

As before, the best way to study is to review the problem sets and the questions below, paying special attention to homework problems on which you didn't get full credit (I have some extra copies of the solutions if you need them). It's also worth looking in your notes for any points that I may have stressed over and over again in class—such things tend to wind up on exams.

Office hours

In addition to my usual office hours next week, I'll be around for most of the afternoon on Thursday (after lecture), at least until 6pm.

Review problems

Here are some problems to practice with:
From Simmons:

- Chapter 5 Additional problems (pp. 188–189): #3, 19, 23, 33
- Chapter 6 Additional problems (pp. 217–218): #7, 9, 11
- Chapter 7 Additional problems (pp. 254–257): #19, 21, 23, 27, 35, 39
- Chapter 10 Additional problems (pp. 375–378): #1, 7, 31, 53, 79, 83, 85, 87, 91, 97, 101, 103, 107, 111, 121, 127, 131, 133, 139, 143

Thinking questions

1. If $F(x)$ is a differentiable function such that $F'(x) = e^{-x^2}$ and $F(1) = 0$, what is $F(2)$? Write the answer in a form that would be suitable for computing $F(2)$ with a graphing calculator (you might not be able to do it by hand).

2. The following differential equation is a special case of the *logistic equation*:

$$\frac{dy}{dx} = y(8 - y).$$

   Draw the corresponding direction field (see the applets on the website to remind yourself what a direction field is). Now use it to answer the following question *without solving the equation*: if $y(x)$ is a solution with $y(0) = 1$, what are

$$\lim_{x \to \infty} y(x) \qquad \text{and} \qquad \lim_{x \to -\infty} y(x)?$$

   For good measure, solve the equation now and see if you were right.

3. What is $\displaystyle\int_0^4 \frac{dx}{x^2 - 4x + 3}$?