18.01 SPRING 2005
PROBLEM SET 11
DUE TUESDAY, MAY 3

The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can't be sure that you didn't just copy the answers from someone else, and there's no way to give partial credit.
- You're free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.
- You can either hand in your solutions in class by the due date, or slip them through the slot in my office door (2-172) by 11:59pm that night.

Reading

Notes INT, Simmons 12.4, 13.1–13.4.

Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but *do not hand them in.* The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- *Tu 4/26/05:* 6B-1 to 6B-3, 6B-7*
- *Th 4/28/05:* 7A-1*, 7A-2, 7A-3, 7A-4*
- *Fr 4/29/05:* 6C-1, 6C-2abe

Graded problems, Part A [70 pts total]

From Simmons:

- 12.4 #2, 4, 6, 8, 12, 14 [3 pts each], 18 [4 pts], 24a [3 pts]
- 13.2 #2 [3 pts], 3 [10 pts], 4de [6 pts]
- 13.3 #1 [2 pts], 2bgkl [12 pts], 10 [12 pts]

Graded problems, Part B [18 pts total]

1. Consider the infinite series
$$\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}.$$
   (a) [2 pts] Use partial fractions to rewrite the terms of this series.
   (b) [5 pts] If you add the first several terms of the series using the partial fractions trick of part (a), you'll notice some cancellation happening. With this in mind, write down a formula for the partial sums
$$S_N = \sum_{n=1}^{N} \frac{1}{n^2+4n+3}$$
   when N is sufficiently large. (It's okay if your formula doesn't apply when N is small.)
   (c) [3 pts] Compute the sum of the series: recall that by definition, this sum is the limit of S_N as $N \to \infty$.

2. The *Cantor set*, named after the mathematician Georg Cantor, is constructed as follows: we start with the closed interval $[0, 1]$, and remove the open interval $\left(\frac{1}{3}, \frac{2}{3}\right)$. That leaves the two closed intervals $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$. Next we remove the open middle third of each of these two intervals. Four closed intervals will then remain, and again we remove the open middle third of each of them. We continue this procedure indefinitely, at each step removing the open middle third of each of the closed intervals that remains. The Cantor set consists of all numbers in $[0, 1]$ that are never removed at any step in this process, i.e. everything that's left after infinitely many open intervals are removed.

   (a) [5 pts] Show that the lengths of all the open intervals removed add up to 1.

   (b) [3 pts] Despite that, there are still numbers in the Cantor set, in fact infinitely many. Give examples of a few.