18.01 SPRING 2005
PROBLEM SET 2
DUE TUESDAY, FEBRUARY 15

The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can't be sure that you didn't just copy the answers from someone else, and there's no way to give partial credit.
- You're free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.
- You can either hand in your solutions in class by the due date, or slip them through the slot in my office door (2-172) by 11:59pm that night.

Reading

Simmons 3.4–3.6, 8.1–8.2, 8.3 (skip Examples 2 and 3), 8.4 (through Example 1), 9.1–9.2, 9.4. Notes X.

Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but *do not hand them in*. The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- *Tu 2/8/05:* 1F-3*, 1F-4, 1F-8; Simmons 3.5 #1, 7, 15, 29
- *Th 2/10/05:* 1G-1, 1G-2*, 1G-4; Simmons 3.4 #7, 13; 3.6 #7
- *Fr 2/11/05:* 1H-1, 1H-3, 1H-5, 1I-1, 1I-3, 1I-4

Graded problems, Part A [75 pts total]

From Simmons:

- 3.4 #2, 4, 14, 36 [2 pts each]
- 3.5 #2, 8, 24, 26, 36, 44 [3 pts each]
- 3.6 #8 [4 pts]
- 8.3 #4, 6, 10 [2 pts each], 26 [4 pts]
- 8.4 #2 [22 pts], 4(c)–(f) [8 pts], 18 [5 pts]

Graded problems, Part B [15 pts total]

1. In lecture we used the product rule and an induction argument to prove the power rule
$$\frac{d}{dx}x^n = nx^{n-1} \tag{1}$$
for all positive integers n. The formula is actually true for *all real* n. In this problem we complete the proof when n is any positive or negative rational number; recall that a number is *rational* if it can be expressed as a fraction p/q for some pair of integers p and q.
    (a) [5 pts] Assume p and q are positive integers and let $n = p/q$. By definition then, $x^n = \sqrt[q]{x^p}$. Use the positive integer case of Formula (1) together with implicit differentiation to prove that
    $$\frac{d}{dx}x^{p/q} = \frac{p}{q}x^{(p/q-1)}.$$

Note that when you use Formula (1) for this, you may not assume the exponent is anything more general than a positive integer; more would be circular reasoning.

(b) [5 pts] Now that you've proved the power rule for all positive rational n, use this result and implicit differentiation to show that it's true for *negative* rational n as well. That is, show that if $n > 0$,

$$\frac{d}{dx}x^{-n} = -nx^{-n-1}.$$

This time you're allowed to use Formula (1) for any *positive* fraction $n = p/q$, but not for negative n, since that's the case you're trying to prove.

2. [5 pts] Compute the 174th derivative of $f(x) = \sin ax$, where a is a constant. Explain your answer.