18.01 SPRING 2005
PROBLEM SET 2
SOLUTIONS

Graded problems, Part A

See attached photocopies.

Graded problems, Part B

1. (a) Let $y = x^{p/q}$, so by definition $y^q = x^p$. Differentiate both sides of this with respect to x, obtaining

$$qy^{q-1}\frac{dy}{dx} = px^{p-1}.$$

Note that we're allowed to use the power rule for this because both p and q are assumed to be positive integers. Solving for $\frac{dy}{dx}$, this becomes

$$\frac{dy}{dx} = \frac{p}{q}\frac{x^{p-1}}{y^{q-1}} = \frac{p}{q}\frac{x^{p-1}}{(x^{p/q})^{q-1}} = \frac{p}{q}\frac{x^{p-1}}{x^{\frac{p}{q}(q-1)}} = \frac{p}{q}x^{p-1-(p-\frac{p}{q})} = \frac{p}{q}x^{p/q-1}.$$

(b) Assume n is a positive rational number and $y = x^{-n}$. Then $yx^n = 1$, and differentiating both sides with respect to x yields

$$\frac{dy}{dx}x^n + ynx^{n-1} = 0.$$

Here we've used the power rule to differentiate x^n since n is positive. Solving for $\frac{dy}{dx}$, we find

$$\frac{dy}{dx} = -n\frac{yx^{n-1}}{x^n} = -n\frac{x^{-n}x^{n-1}}{x^n} = -nx^{-n-1}.$$

The upshot is that from now on you can feel free to use the power rule without worrying whether n is positive, negative, rational or whatever!

2. The 4th derivative of $\sin x$ is itself, thus the same is true for $\frac{d^n}{dx^n}\sin x$ whenever n is any multiple of 4. For $\sin ax$ we have to use the chain rule, which introduces a new factor of a in front every time we differentiate, i.e. $\frac{d^2}{dx^2}\sin ax = -a^2 \sin ax$ and so forth; in particular, $\frac{d^n}{dx^n}\sin ax = a^n \sin ax$ if n is divisible by 4. Since 172 is divisible by 4, we have

$$\frac{d^{174}}{dx^{174}}\sin ax = \frac{d^2}{dx^2}\frac{d^{172}}{dx^{172}}\sin ax = \frac{d^2}{dx^2}a^{172}\sin ax = -a^{174}\sin ax.$$