**18.01 SPRING 2005**
**PROBLEM SET 4**
**DUE TUESDAY, MARCH 8**

### The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can't be sure that you didn't just copy the answers from someone else, and there's no way to give partial credit.
- You're free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.
- You can either hand in your solutions in class by the due date, or slip them through the slot in my office door (2-172) by 11:59pm that night.

### Reading

Simmons 6.1–6.7, and review Notes MVT.

### Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but *do not hand them in*. The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- *Th 3/03/05:* 3B-2, 3B-3, 3B-5
- *Fr 3/04/05:* 3C-1 through 3C-6

### Graded problems, Part A [37 pts total]

*From Simmons:*

- 6.6 #2, 4, 6, 16, 22, 26, 28 [3 pts each]
- 6.7 #7, 8, 9, 10 [4 pts each]

### Graded problems, Part B [19 pts total]

1. A generalization of the mean value theorem says that if $f''(x)$ exists for all $x$ and $a < b$, then there exists a number $c \in (a, b)$ such that

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(c)}{2}(b-a)^2. \qquad (1)$$

A useful application of this formula will be given in the next problem; the goal of this one is to prove the formula.

(a) [2 pts] Show that there is a unique number $k$ such that

$$f(b) = f(a) + f'(a)(b-a) + k(b-a)^2.$$

(This is easy: just solve for $k$.) The goal from now on will thus be to show that $k = f''(c)/2$ for some $c \in (a, b)$.

(b) [3 pts] Let $g(x) = f(x) - f(a) - f'(a)(x-a) - k(x-a)^2$. Use Rolle's theorem to show that there exists a number $c_1 \in (a, b)$ such that $g'(c_1) = 0$.

(c) [2 pts] Compute $g'(a)$.

(d) [3 pts] Show that there exists a number $c \in (a, c_1)$ such that $g''(c) = 0$.

(e) [2 pts] Conclude that $k = f''(c)/2$.

2. (a) [2 pts] Write down the linear approximation to $\sqrt[3]{1+2x}$ for $x$ near 0. (Recall that you can do this without differentiating anything!)

   (b) [5 pts] Use Formula (1) from the previous problem to determine an error estimate for your linear approximation from part (a), assuming $0 \le x \le 1/8$. That is, find a number $M > 0$ such that the magnitude of the difference between $\sqrt[3]{1+2x}$ and its linear approximation does not exceed $M$ when $x \in [0, 1/8]$.