18.01 SPRING 2005
PROBLEM SET 7
DUE TUESDAY, APRIL 5

Achtung! Midterm 2 is coming up: Friday, April 8. More details will be announced by the end of this week.

Special note about the Midterm 1 take-home

If you haven't gotten back your take-home addendum to Midterm 1, you should get it from me ASAP. It's graded either with a "check plus" or with an instruction to redo certain problems. In the latter case, you must write up new solutions to those problems and give them back to me before the second midterm. It's important that the new solutions be **clearly written** and **correct**—to that end, you're encouraged to come and speak with me in my office before completing them.

The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can't be sure that you didn't just copy the answers from someone else, and there's no way to give partial credit.
- You're free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.
- You can either hand in your solutions in class by the due date, or slip them through the slot in my office door (2-172) by 11:59pm that night.

Reading

Simmons 9.5, 9.7, 10.3–10.6.

Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but *do not hand them in.* The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- *Tu 3/29/05:* 5A-1, 5A-3, 5A-5*, 5C-2 to 5C-7, 5C-9, 5C-11*
- *Th 3/31/05:* 5D-1 to 5D-8
- *Fr 4/1/05:* 5D-10 to 5D-14, 5E-1, 5E-10a, 5E-11*

Graded problems, Part A [58 pts total]

From Simmons:

- 9.5 #2, 6, 10 [2 pts each], 14 [4 pts]
- 9.7 #26 [3 pts], 27 (use implicit differentiation) [3 pts]
- 10.3 #2, 6, 12, [3 pts each], 26 [5 pts]
- 10.4 #2, 4, 8, 18, 20 [4 pts each]
- 10.5 #10, 16 [4 pts each]

Graded problems, Part B [13 pts total]

1. Consider the indefinite integral
$$\int \frac{dx}{x^2-1}.$$
   (a) [3 pts] Compute it by the method of partial fractions.
   (b) [3 pts] Compute it by trigonometric substitution. (You might find it helpful to look up $\int \sec u \, du$ or $\int \csc u \, du$ on the inside back cover of Simmons.)
   (c) [2 pts] If both answers are correct, they need not be equal, but should differ only by a constant. Verify that this is the case.

2. [5 pts] Consider the following calculation: we want to compute $\displaystyle\int_{-2}^{2} \frac{dx}{\sqrt{x^2-1}}$. Substituting $x = \sec\theta$, we have $dx = \sec\theta\tan\theta \; d\theta$ and $\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \tan\theta$, so
$$\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{\sec\theta\tan\theta \; d\theta}{\tan\theta} = \int \sec\theta \; d\theta = \ln|\sec\theta + \tan\theta| = \ln\left|x + \sqrt{x^2-1}\right|.$$
   To be sure we've done this right, we can check that $\displaystyle\frac{d}{dx}\ln\left|x+\sqrt{x^2-1}\right| = \frac{1}{\sqrt{x^2-1}}$. Hence
$$\int_{-2}^{2} \frac{dx}{\sqrt{x^2-1}} = \ln\left|x+\sqrt{x^2-1}\right|\Bigg|_{-2}^{2} = \ln\left|2+\sqrt{3}\right| - \ln\left|-2+\sqrt{3}\right| = \ln\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right).$$
   This answer is utter nonsense. Explain why.