18.01 SPRING 2005
PROBLEM SET 9
DUE WEDNESDAY, APRIL 20

Note: this is due on Wednesday since Tuesday is a holiday.

The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can't be sure that you didn't just copy the answers from someone else, and there's no way to give partial credit.
- You're free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.
- You can either hand in your solutions in class by the due date, or slip them through the slot in my office door (2-172) by 11:59pm that night.

Reading

Simmons 17.1, 7.5, 7.6, 16.1–16.3, 17.2 (just for fun).

Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but *do not hand them in.* The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- *Tu 4/12/05:* 4E-3, 4E-4, 4E-6, 4E-8*; Simmons 17.1 #11*
- *Th 4/14/05:* 4F-1, 4F-4, 4F-6, 4F-8*, 4G-1, 4G-4, 4G-5*
- *Fr 4/15/05:* 4H-1, 4H-2a(i), 4H-3*

Graded problems, Part A [52 pts total]

From Simmons:

- 17.1 #4, 8 [3 pts each]
- 7.5 #2 [3 pts], 10 [5 pts]
- 7.6 #2, 4 [3 pts each]
- 16.1 #2(a,b,c) [2 pts each], 6 [6 pts]
- 16.2 #2 [12 pts], 4(i,k) [4 pts each]

Graded problems, Part B [20 pts total]

1. Given a constant $a > 0$, the graph of the equation

$$x^3 + y^3 = 3axy$$

is called the *folium of Descartes.* A graph is shown in Simmons, P. 592 Figure 17.11. The goal of this problem is to justify and elucidate that picture (see also Problem #16 on the same page).

   (a) [4 pts] Introduce the parameter $t = y/x$ and show that the equation above is then equivalent to the parametric equations

$$x(t) = \frac{3at}{1+t^3}, \qquad y(t) = \frac{3at^2}{1+t^3}$$

   for $t \neq -1$.

(b) [2 pts] Use the parametric equations from part (a) to show that the line $x + y = -a$ is an asymptote, by showing that $x(t) + y(t) \to -a$ as $t \to -1$.

(c) [5 pts] Identify all values of t for which the tangent line to the curve $(x(t), y(t))$ is horizontal or vertical. Find the coordinates of the corresponding points (x, y) on the curve.

(d) [2 pts] Show that the curve is symmetric about the diagonal line $y = x$.

(e) [7 pts] Plot the curve, with all of the points from part (c) labelled, as well as the asymptote of part (b). Put in a few arrows to indicate how the curve is traced out as t increases. Label also the portions of the curve that are traced as $t \to +\infty$ and as $t \to -\infty$. Finally, label the portions that are traced as $t \to -1$ from above and from below; both of these portions approach the asymptote, but in different places!