18.950—DIFFERENTIAL GEOMETRY
SPRING 2007

Practical details

Instructor: Chris Wendl, office 2-169, wendlc@math.mit.edu, 617-253-4057

Office hours: Tuesday and Thursday 4:15–5:15pm, and other times by appointment. Feel free to stop by my office anytime, but be aware that I'm often not there; best to call ahead.

Lectures: Tuesday and Thursday 11am–12:30pm in 2-102

Course web page: http://math.mit.edu/18.950/

Course description

An introduction to differential geometry. Calculus on n-dimensional manifolds, vector fields, integration, Stokes' theorem. Connections and curvature on fiber bundles and vector bundles, with applications to Riemannian geometry and physics.

Prerequisites

18.100; 18.101 or 18.901 strongly recommended.

To be precise: you are expected to understand fundamental concepts of mathematical analysis such as metric spaces and open sets, with sufficient "mathematical maturity" to write readable proofs. A good understanding of basic linear algebra (vector spaces, matrices, linear maps, bases) and differentiable calculus for maps $\mathbb{R}^n \to \mathbb{R}^m$ (the derivative as a linear map, chain rule and inverse function theorem) is also important, but will be reviewed (quickly!) in the first two lectures. If you've taken 18.101 or perused the first two chapters of either Spivak's *Calculus on Manifolds* or Munkres' *Analysis on Manifolds*, you're likely in good shape—if not, you might want to take some time with one of those books this weekend.

Required texts

We will not follow either of these religiously, but together they contain everything we'll cover in the course, and will serve as important supplements to the lectures. Specific reading assignments will be specified each week.

- M. Spivak: *A Comprehensive Introduction to Differential Geometry, Volume I, third edition.* This is a unique, clear, entertaining and beautifully written book, never to be faulted for lack of detail. We will cover the majority of Chapters 1–5, 7 and 8 during the first unit of the course (weeks 1–5), though I'll sometimes recommend skipping certain bits unless you have an abundance of free time.
- C. Wendl: *Notes on Bundles and Connections*, available for download from the course web page or as printouts in lecture. These notes are fairly detailed and will serve as the main text from week 6 onward. They also contain some topics that we will not cover, which you may consider optional reading.

Recommended texts and background/supplemental reading

Here are some good books you can consult whenever you feel you need more than what's listed above.

- M. Spivak: *A Comprehensive Introduction to Differential Geometry, Volume II, third edition.* This is on reserve in Hayden Library; it covers many of the topics covered in the lecture notes, from a slightly different perspective.
- M. Spivak: *Calculus on Manifolds.* A beautiful little book that covers the basics of manifolds and differential forms (thus overlapping somewhat with the Differential Geometry book listed above), but also more fundamental topics in advanced calculus of several variables, which may be considered prerequisites for this course.
- J. Munkres: *Analysis on Manifolds.* Similar to Spivak's book of similar title but larger, and particularly recommended for its review of basic linear algebra and calculus of maps $\mathbb{R}^n \to \mathbb{R}^m$.
- R. Creighton Buck: *Advanced Calculus.* Another good book on calculus of differentiable maps $\mathbb{R}^n \to \mathbb{R}^m$, particularly when n and $m \leq 3$.
- S. Carroll: *Lecture Notes on General Relativity*, available for free download at `http://pancake.uchicago.edu/~carroll/notes/`. For those interested in physics, these notes include a very clear introduction to manifolds and curvature, with Einstein's theory of gravitation as motivation. It's also good for learning to understand the tensor index notation that physicists are so very fond of.

Problem sets

Problem sets of short to moderate length will be assigned approximately once every one or two weeks, usually due on Thursdays in lecture or by 5pm at my office. The lengths will vary and so will the number of points each is worth, which will be stated explicitly on each assignment. You may discuss the problems with your fellow students but must write up the solutions independently.

Late homework may be accepted under exceptional circumstances, but *not more than once* in the semester, and *only if you contact me beforehand* to explain the situation.

Exams

There will be two *take-home* midterms, which will take the place of problem sets in approximately the fifth and tenth weeks of the semester. These will be somewhat more involved than the problem sets and will be graded more carefully—use of books and other resources will be allowed, but discussion with other students will *not*.

The final exam will be an open-book test with a scheduled 3-hour time slot during finals period—I will try to write the test so that it is doable in half the available time.

Grading

Problem sets (30% total), two take-home midterms (25% each), final exam (20%).

Syllabus

The following is subject to change, but we'll try our best to stick to this plan.

The course is divided roughly into three units:

I. **Fundamentals of manifolds** (weeks 1–5)

II. **Vector bundles and connections** (weeks 5–10)

III. **Applications** (week 11–end)

Week-by-week breakdown:

1. Differentiable manifolds, implicit function theorem, examples.
2. Tangent maps, orientability, vector fields, flows, Lie bracket.
3. Tensors, index notation, differential forms, Poincaré lemma, Lie derivative.
4. Integration, Stokes' theorem, low-dimensional examples.
5. Basic Lie groups, exponential map, vector bundles and sections.
6. Bundle metrics, orientation, structure groups, fiber bundles.
7. Parallel transport, connections, covariant derivatives, compatibility.
8. Connections on tangent bundles, torsion and symmetry, Riemann normal coordinates.
9. Levi-Cività connection, geodesics, Riemannian manifolds.
10. Integrability and the Frobenius theorem, curvature.
11. Locally flat manifolds, Gaussian curvature.
12. Euler characteristic and the Gauss-Bonnet theorem for surfaces.
13. (if time permits) Introduction to general relativity, gauge theory.