Course description and syllabus

General information

Instructors: Prof. Chris Wendl (lectures)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.301
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Office hour: Wednesdays 15:00–16:00

Dr. Marc Kegel (problem sessions)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.318
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Website: www.mathematik.hu-berlin.de/~wendl/Sommer2022/Diffgeo2/

Moodle: moodle.hu-berlin.de/course/view.php?id=110684
enrolment key: Riemann
The moodle will be used mainly for communication: you must join it if you want to receive occasional important announcements about the course via e-mail, and you can also use the moodle forum to discuss or ask questions about homework problems. Essential course materials such as problem sets and lecture notes will be posted on the course website rather than the moodle.

Lectures: Wednesdays 11:15–12:45 in 1.114 (Rudower Chaussee 25)
Thursdays 11:15–12:45 in 1.114 (Rudower Chaussee 25)

Problem sessions: Wednesdays 9:15–10:45 in 1.114 (Rudower Chaussee 25)

Language: This course will be taught in English.

Prerequisites: The contents of the HU's course *Differentialgeometrie I* as taught in Winter Semester 2021–22: in particular, students will be assumed familiar with the basic theory of smooth manifolds, including vector fields and the Lie bracket, tensors, differential forms, integration and Stokes' theorem. Knowledge of vector bundles and connections will be helpful, but these topics will be reviewed quickly at the beginning. Some knowledge of functional analysis and/or algebraic topology will also be helpful in certain portions of the course, but not strictly necessary.

Course description

The course will survey a selection of intermediate-level topics in differential geometry, including Lie groups and Lie algebras, theorems about curvature in higher-dimensional Riemannian manifolds, Hodge theory and harmonic forms on Riemannian manifolds, principal fiber bundles and fundamentals of gauge theory. If time permits, we will also discuss characteristic classes, Kähler manifolds and holonomy.

Syllabus

The main portion of the course will be divided into five units, with two additional units to be included at the end of there is enough time. The following plan is tentative and subject to change.

1. Some (pseudo-)Riemannian geometry
    - Review of vector bundles and connections
    - Geodesics, exponential map, Hopf-Rinow theorem
    - Riemann curvature tensor and its symmetries
    - Ricci and scalar curvature, Gauss and sectional curvature
    - Jacobi vector fields, Cartan-Hadamard theorem
2. Lie groups
    - Left-invariant vector fields, the Lie algebra of a Lie group, exponential map
    - Closed subgroups are Lie subgroups
    - Bi-invariant metrics, Haar measure, geodesics on Lie groups
    - Smooth group actions, slice theorem, homogeneous spaces
3. Principal fiber bundles
    - Fiber bundles and their structure groups
    - Principal and frame bundles, associated bundles
    - Connections and curvature on (principal) fiber bundles
    - Spin structures and Dirac operators
    - Introduction to mathematical gauge theory
4. Riemannian manifolds with symmetry
    - Killing vector fields, affine transformations and isometry groups
    - Constant sectional curvature and local isometries
    - Locally symmetric spaces
    - Classification of Riemannian space forms
5. Hodge theory
    - Survey of Fourier series and Sobolev spaces
    - Elliptic operators and local regularity
    - The Hodge Laplacian is a Fredholm operator
    - Hodge decomposition theorem

And if time permits:

6. Chern-Weil theory and characteristic classes
7. Kähler manifolds, Einstein metrics and holonomy

Literature

Lecture notes to accompany this course will be posted regularly on the course website. For alternative treatments of the same or similar material, the following sources are recommended:

- John M. Lee, *Introduction to Smooth Manifolds*, second edition, Springer GTM 2012
  (online access available via the HU library)

- John M. Lee, *Riemannian Manifolds An Introduction to Curvature*, Springer GTM 1997
(online access available via the HU library)
- Michael Spivak, *A Comprehensive Introduction to Differential Geometry, Volumes I–V*, 3rd edition with corrections, Publish or Perish 2005
(available in the HU library, Freihandbestand)
- Frank W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer GTM 1983
(online access available via the HU library)
- Helga Baum, *Eine Einführung in die Differentialgeometrie*, lecture notes available at
`https://www.mathematik.hu-berlin.de/~baum/Skript/diffgeo1.pdf`

Exam and problem sets

Grades in the course will be determined by a short **oral exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). In the exam, you will need to be able to write down the main definitions in the course, discuss their meaning and significance (with reference to examples where appropriate), and describe the most important applications of the major theorems and the main ideas behind their proofs. The range of topics covered on the exam will be specified ahead of time in consultation with each individual student; in this way, students with insufficient backgrounds in functional analysis or topology will have the option to exclude the relevant topics from the exam.

Problem sets will be distributed and posted on the course website every Thursday, and solutions discussed in the problem session on the following Wednesday. The problem sets will not be graded, but it is **strongly recommended** that you at least think through every problem before the problem session each week, since this is the single best way to ensure that you are keeping up with the material in the course.

There will also be a special homework assignment midway through the semester, the so-called **take-home midterm**, which you will have two weeks to work on and can submit for a grade. The midterm is voluntary, but your score can be used to boost your final exam grade according to the following rule:

- Midterm 60%–79% $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,7$ or $1,7 \rightsquigarrow 1,3$ etc.
- Midterm 80%–100% $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,3$ or $1,7 \rightsquigarrow 1,0$ etc.