Topology II — Humboldt-Universität zu Berlin
C. Wendl, F. Schmäschke — Winter Semester 2018–19

Course description and syllabus

General information

Instructors: Prof. Chris Wendl (lectures)
HU Institute for Mathematics (Rudower Chaussee 25), Room 1.301
`wendl@math.hu-berlin.de`
Office hour: Tuesdays 15:00–16:00

Dr. Felix Schmäschke (problem classes)
HU Institute for Mathematics (Rudower Chaussee 25), Room 1.303
`felix.schmaeschke@math.hu-berlin.de`
Office hour: Tuesdays 10:00–11:00

Course webpage: `http://www.mathematik.hu-berlin.de/~wendl/Winter2018/Topologie2/`

Lectures: Wednesdays 11:00–13:00 in 1.013 (Rudower Chaussee 25)
Fridays 9:00–11:00 in 1.013 (Rudower Chaussee 25)

Problem classes: Wednesdays 13:00–15:00 in 1.013 (Rudower Chaussee 25)

Language: The course will be taught in English.

Prerequisites: The contents of the HU's course *Topologie I* as taught in Summer Semester 2017 or 2018, including: essentials of point-set topology, fundamental group and covering spaces, simplicial complexes, basics of singular homology.

Students who have not previously seen the main definitions of singular homology are advised to read through the last five lectures of the notes from last semester's *Topologie I* class, available at `https://www.mathematik.hu-berlin.de/~wendl/Sommer2018/Topologie1/lecturenotes.pdf`
We will quickly review this material in the first week.

Course description

This is a course in algebraic topology for students with background knowledge of the material in *Topologie I* as described above under **Prerequisites**. We will develop the singular homology and cohomology functors in depth, with emphasis on the homology of CW-complexes and manifolds, and also their role within the wider context of axiomatic homology theories and their relationship with higher homotopy groups. The tentative program includes as many of the following topics as will fit into one semester:

1. Introduction to categories and functors
2. Review of singular homology (homotopy invariance, excision, long exact sequence of the pair)
3. Reduced homology and computation of $H_*(S^n)$
4. Mayer-Vietoris sequence and applications
5. Degree of a map
6. Singular cohomology

7. The Eilenberg-Steenrod axioms for homology and cohomology theories
8. Direct and inverse limits
9. Brief sketch of alternative homology/cohomology theories (Čech and Alexander-Spanier)
10. Axiomatic computation of homology/cohomology for CW-complexes
11. The Lefschetz fixed point theorem
12. Universal coefficient theorem
13. Cross product, cup product and the Künneth formula
14. Topological manifolds, fundamental classes and Poincaré duality
15. Sketch of homological intersection theory for submanifolds
16. Higher homotopy groups
17. Serre fibrations and the homotopy exact sequence
18. Hurewicz homomorphism $\pi_k(X) \to H_k(X)$
19. Whitehead's theorem on weak homotopy equivalences

Literature

Almost everything we will discuss in this course is contained in at least one of the following two books:

- Glen Bredon, *Topology and Geometry*, Springer GTM 1993
  (online access available via the HU library)
- Allen Hatcher, *Algebraic Topology*, Cambridge University Press 2002
  (also freely downloadable from the author's homepage:
  `https://www.math.cornell.edu/~hatcher/AT/ATpage.html`)

We will not follow either book precisely, but will follow Bredon slightly more closely than Hatcher. Here are some other standard algebraic topology books that overlap heavily with each of these:

- James W. Vick, *Homology Theory*, Springer GTM 1994
  (online access available via the HU library)
- R. Stöcker und H. Zieschang, *Algebraische Topologie - Eine Einführung*, Teubner 1994
  (available in the HU library, Freihandbestand)

Finally, the following book is a classic which I cannot recommend as a textbook for learning the material, but its importance as a historical document earns it a place on this list:

- Samuel Eilenberg and Normal Steenrod, *Foundations of Algebraic Topology*, Princton U. Press 1952
  (available in the HU library, Freihandbestand)

Exam and problem sets

Grades in the course will be determined by a short **oral exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). In the exam, you will need to be able to write down the main definitions in the course, discuss their meaning and significance (with reference to examples where appropriate), and describe the most important applications of the major theorems and the main ideas behind their proofs.

There will be one graded assignment midway through the semester, a so-called **take-home midterm**, which you will have two weeks to work on. Achieving a score of 75% or better on the take-home midterm can boost your final exam grade by one notch, i.e.

- $\geq 75\%$ on midterm = (2,0 $\to$ 1,7 or 1,7 $\to$ 1,3 etc.)

There will also be ungraded **problem sets** handed out every Wednesday and discussed in the problem class on the following Wednesday.

Werbung

Students may also be interested in the following courses being taught this semester:

- *Differentialgeometrie I* (Prof. H. Baum): If you are seriously interested in topology and geometry but have not yet taken this course, you should take it as soon as possible.
- Proseminar *Differentialtopologie* (Dr. M. Kegel): This seminar should be perfect for Bachelor students who took *Topologie I* last semester, but it is also open to enthusiastic students in their third semester. It will be based mostly on Milnor's classic book *Topology from the Differentiable Viewpoint*.
- *Symplectic Topology and Holomorphic Curves*: I will be teaching this special topics course, aimed at Masters students who already have some background in differential geometry and are not afraid of analysis.
- Seminar *Floer Homologie* (Prof. K. Mohnke): This seminar is an ideal accompaniment to my special topics course mentioned above, but also requires knowledge of the theory of smooth manifolds and functional analysis.
- *Lorenzgeometrie und mathematische Relativitätstheorie* (Dr. O. Müller): Ideal for Masters students who have some background in differential geometry and are interested in physics.