Course description and syllabus

General information

Instructors: Prof. Chris Wendl (lectures)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.301
`wendl@math.hu-berlin.de`
Office hour: Fridays 11:00–12:00

Jacek Rzemieniecki (problem sessions)
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Website: `www.mathematik.hu-berlin.de/~wendl/Winter2025/FunkAna/`

Moodle: `moodle.hu-berlin.de/course/view.php?id=136548`
Enrolment key: `Hilbert`

Lectures: Mondays 11:00–12:30 in 1.013 (Rudower Chaussee 25)
Thursdays 15:15–16:45 in 1.013 (Rudower Chaussee 25)

Problem sessions: Thursdays 13:15–14:45 in 3.007 (Rudower Chaussee 25)

Language: As a *BMS Basic Course*, this course will be offered in English unless all students prefer to hear it in German. This will be decided in the first lecture.

Prerequisites: The contents of the HU's courses *Analysis I–III* and *Lineare Algebra und Analytische Geometrie I–II*, as well as the analytical part of the course *Algebra und Funktionentheorie* (i.e. basic complex analysis).

All students should in particular be familiar with the fundamentals of measure theory (including Fubini's theorem and the completeness of the L^p spaces).

Course description

This is a course on *linear* functional analysis, which can be defined as the study of continuous linear maps between infinite-dimensional topological vector spaces (mainly Banach and Hilbert spaces). The most important examples of such infinite-dimensional vector spaces are function spaces, which often arise in applications e.g. as solution spaces for partial differential equations. The contents of this course should therefore be seen as essential preparation for any course (either in analysis, applied mathematics, differential geometry or mathematical physics) dealing with PDEs.

Syllabus

The course will be divided into four segments:

I. **Fundamentals** (weeks 1–3)

II. **Real analysis and L^p-spaces** (weeks 4–9)

III. **Abstract Banach spaces** (weeks 10–12)

IV. **Spectral theory** (weeks 13–15)

The following week-by-week schedule is preliminary and subject to change.

1. Banach spaces and continuous/bounded linear operators, the operator norm, basic notions from point-set topology, topological vector spaces, locally convex vector spaces and Fréchet spaces, examples
2. Dual spaces, Zorn's lemma, Hamel bases
3. Basic results on Hilbert spaces: uniform convexity, the Riesz representation theorem, orthonormal bases, orthogonal projections
4. Properties of the L^p spaces on $\mathbb{R}^n$: duality of L^p and L^q for $\frac{1}{p} + \frac{1}{q} = 1$, separability of L^p, weak convergence, the Banach-Alaoglu theorem
5. Convolutions and Young's inequality, approximation by smooth functions, absolute continuity, the Radon-Nikodym theorem, the fundamental theorem of calculus
6. Periodic functions and Fourier series on $L^2(\mathbb{T}^n)$
7. The Fourier transform on Schwartz space and $L^2(\mathbb{R}^n)$
8. The Sobolev spaces $H^k(\mathbb{R}^n)$ and $H^k(\mathbb{T}^n)$
9. Distributions (generalized functions)
10. The Baire category theorem and Hahn-Banach theorem
11. The open mapping theorem, closed subspaces with closed complements
12. Compact operators and Fredholm operators
13. The spectrum of a bounded linear operator on a Hilbert space, polar decomposition
14. Spectral theory for bounded operators
15. Unbounded self-adjoint operators and spectral theory

Literature

Lecture notes for this course will be posted on the course website and updated every week. Otherwise, the course will not follow any particular book, but the following textbooks are highly recommended, especially the book by Reed and Simon.

- Reed and Simon, *Methods of Modern Mathematical Physics I, Functional Analysis*, revised and enlarged edition, Elsevier 2011
  (online access available via the HU library)
- Bühler and Salamon, *Functional Analysis*, AMS 2018
  (preprint version available freely on Salamon's homepage:
  `https://people.math.ethz.ch/~salamon/PREPRINTS/funcana-ams.pdf`)
- Conway, *A Course in Functional Analysis*, Springer 1985
  (online access available via the HU library)

Exam and problem sets

Grades in this course will be determined by a 3-hour **written exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). Books and notes may be consulted during the exam. The exam problems will be conceived so as to be solvable within 2 hours, so that time pressure should not be the decisive factor.

Problem sets will be posted on the course website every Thursday, and solutions discussed in the problem session on the following Thursday. The problem sets will not be graded, but it is **strongly recommended** that you at least think through every problem before the problem session each week, since this is the single best way to ensure that you are keeping up with the material in the course.

There will also be a special homework assignment midway through the semester, the so-called **take-home midterm**, which you will have two weeks to work on. The midterm is voluntary, but your score on it can be used to boost your final exam grade according to the following rule:

- Midterm 60%–79% $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,7$ or $1,7 \rightsquigarrow 1,3$ etc.
- Midterm 80%–100% $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,3$ or $1,7 \rightsquigarrow 1,0$ etc.