Score:	Name:

Midterm 1 - 02/27/03
Calculus I - V63.0121

Masmoudi–Schneiderman–Wendl

Instructions: This midterm should be taken in 70 minutes without text or notes.

1. (15pts.) Compute each derivative:

   a) $f(x) = e^{\sin x}$ ANSWER: $f'(x) = e^{\sin x}(\sin x)' = e^{\sin x}(\cos x)$

   b) $g(x) = e^x \cos(x)$ ANSWER: $g'(x) = e^x \cos(x) + e^x(-\sin(x))$

   c) $h(x) = 8x^5 - 3x^4 + x^3 + 2$ ANSWER: $h'(x) = 40x^4 - 12x^3 + 3x^2$

2. (10 pts.) By writing down an equation or sketching a graph, give an example of a differentiable function whose derivative is *never* equal to zero. ANSWER: E.g. $f(x) = x$, $f'(x) = 1 \neq 0$.

   Then give an example of a differentiable function whose derivative is *always* equal to zero. ANSWER: E.g. $f(x) = 1$, $f'(x) = 0$.

3. (10pts.) Is the following function continuous? Justify your answer.

$$f(x) = \begin{cases} \frac{x^2-4}{x+2} & \text{if } x \neq -2 \\ -4 & \text{if } x = -2 \end{cases}$$

   ANSWER: Yes, for $x \neq -2$ f is a rational function whose denominator is never 0 and for $x = -2$ we have $\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)(x+2)}{x+2} = \lim_{x\to -2}(x-2) = -4 = f(-2)$.

4. (10pts.) Find an equation of the tangent line to the graph of the function $F(x) = 1 - e^x$ at the point $(1, 1-e)$. ANSWER: Eq of tangent line at $(a, F(a))$ is $y - F(a) = F'(a)(x-a)$. For $(a, F(a)) = (1, 1-e)$ we have $y - (1-e) = -e(x-1)$ since $F'(x) = -e^x$ so $F'(1) = -e^1 = -e$.

5. (10pts.) At which points are the tangent lines to the graph of $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 99$ horizontal? ANSWER: Horizontal tangents occur where $y' = 0$. $y' = x^2 - x - 2 = (x-2)(x+1)$ so $y' = 0$ when $x = 2$ and $x = -1$ (that is, at the points $(2, 1/3 \cdot 2^3 - 1/2 \cdot 2^2 - 2 \cdot 2 + 99)$ and $(-1, 1/3 \cdot (-1)^3 - 1/2 \cdot (-1)^2 - 2 \cdot (-1) + 99)$).

6. (5pts.) Compute the limit $\lim_{\theta\to 0} \frac{\sin 2\theta}{3\theta}$. ANSWER: $\lim_{\theta\to 0} \frac{\sin 2\theta}{3\theta} = \lim_{\theta\to 0} \frac{2}{2}\frac{\sin 2\theta}{3\theta} = \frac{2}{3}\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta} = \frac{2}{3} \cdot 1 = \frac{2}{3}$.

7. (20pts.) Differentiate each function:

   a) $f(x) = (3x)(\sin x)(e^x)$ ANSWER:$f'(x) = (3x)'(\sin x \cdot e^x) + (3x)(\sin x \cdot e^x)' = (3)(\sin x \cdot e^x) + (3x)(\cos x \cdot e^x + \sin x \cdot e^x) = 3(\sin x)e^x + 3x(\cos x)e^x + 3x(\sin x)e^x$.

   b) $g(x) = \cos(\sin(e^x))$ ANSWER: $g'(x) = -\sin(\sin(e^x)) \cdot (\sin(e^x))' = -\sin(\sin(e^x)) \cdot \cos(e^x)(e^x)$.

8. (20pts.) Consider the following function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   a) Compute the derivative of f when $x \neq 0$. ANSWER: For $x \neq 0$, $f'(x) = 2x\sin(\frac{1}{x}) + x^2\cos(\frac{1}{x})(\frac{-1}{x^2}) = 2x\sin(\frac{1}{x}) - \cos(\frac{1}{x})$.

   b) Does $f'(0)$ exist? If so, calculuate it. ANSWER: $f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{x^2\sin(\frac{1}{x})}{x} = \lim_{x\to 0} x\sin(\frac{1}{x}) = 0$ by Squeeze Theorem since $-x \leq x\sin(\frac{1}{x}) \leq x$ and both $-x$ and x go to 0 as x goes to 0.

   c) Is f' continuous everywhere? Explain your answer. ANSWER: $f'(x)$ is NOT continuous at $x = 0$ because the limit as x goes to 0 of $f'(x) = 2x\sin(\frac{1}{x}) - \cos(\frac{1}{x})$ does not exist. ($2x\sin(\frac{1}{x})$ goes to 0 but $-\cos(\frac{1}{x})$ oscillates between ± 1.)