Calculus II
Spring 2004
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Sample Calculus I Final

The following is a cross-section of the kinds of problems you should be able to solve, having completed Calculus I. You should work through them and review anything that gives you trouble—most of the problems are slightly harder than what you'd see on an average final exam, but doable. If a significant proportion of this exam perplexes you, then you may want to reconsider taking Calculus II this semester. Please talk to me if you feel uncertain.

The answers are on the next page.

1. Evaluate each definite or indefinite integral:
   (a) $\displaystyle\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta\cos(\theta^2)\ d\theta.$
   (b) $\displaystyle\int 2x\sin x\ dx$
   (c) $\displaystyle\int_2^3 \frac{dx}{x^2-1}$
2. Differentiate:
   (a) $y = x(\ln x - 1)$
   (b) $y = x^x$
3. Consider the function
$$f(x) = e^{-\frac{1}{x+1}}.$$
   (a) Find all horizontal and vertical asymptotes, if any.
   (b) Find the intervals of decrease and increase.
   (c) Find all local maximum and minimum values, if any.
   (d) Find the intervals of concavity and inflection points.
   (e) Use the information from parts a)-d) to sketch the curve.
4. Consider the relation
$$\frac{y}{x-y} = x^2 + 1.$$
   This defines y implicitly as a function of x.
   (a) Using implicit differentiation, find an expression for $\dfrac{dy}{dx}$ in terms of x and y.
   (b) Find an equation for the tangent line through the point $(1, 2/3)$.
5. Find the derivative of the function
$$f(x) = \int_1^{x^3} \sqrt{t}\cdot\sin(t)\ dt.$$

Answers

1. (a) $-1/2$
   (b) $-2x\cos x + 2\sin x$
   (c) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$
2. (a) $\ln x$
   (b) $x^x(1+\ln x)$
3. (a) There's a vertical asymptote at $x=-1$: $f(x)$ approaches $+\infty$ from the left, and 0 from the right. The horizontal line $y=1$ is also an asymptote, with $f(x)$ approaching 1 as x goes to either $+\infty$ or $-\infty$.
   (b) The function is always increasing, except at $x=-1$ where it's undefined.
   (c) There are no local maxima or minima.
   (d) $x=-1/2$ is an inflection point. The function is concave up for $x<-1$ and $-1<x<-1/2$. It's concave down for $x>-1/2$.
   (e) Do it yourself!
4. (a) $\dfrac{dy}{dx} = \dfrac{3x^2-2xy+1}{x^2+2}$
   (b) $y - \dfrac{2}{3} = \dfrac{8}{9}(x-1)$
5. $f'(x) = 3x^2\sqrt{x^3}\sin(x^3)$