Calculus II
Spring 2004
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Extra Credit Assignment, due Tuesday April 13

A complete and correct answer to the following will earn you the equivalent of an extra 100-point homework assignment to be factored into your homework average. **Please hand this in on a separate sheet from Homework #8.**

Most functions one encounters from day to day are equal to their Taylor series, at least in some interval. Writing down an example of one that does *not* equal its Taylor series requires some cleverness. The following is such an example:

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

This function is continuous and has derivatives of all orders for all x. This fact is fairly obvious for all $x \neq 0$, but at 0 it takes more work to prove it. Start by proving that $f(x)$ is continuous at 0. (Recall: this means $f(0) = \lim_{x\to 0} f(x)$.) Then compute all the derivatives at 0 (you'll need the definition of the derivative), and use them to write down the Taylor series for $f(x)$ about 0. You'll see that the series clearly does not equal the function, except perhaps at $x = 0$.