Calculus II
Spring 2004
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Final Exam: Formula Sheet

1. Some useful antiderivatives:

$$\int \sec^2 x\ dx = \tan x + C \qquad \int \csc^2 x\ dx = -\cot x + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\ dx = \ln|\csc x - \cot x| + C$$

$$\int \sec x \tan x\ dx = \sec x + C \qquad \int \csc x \cot x\ dx = -\csc x + C$$

2. Triogonometry:
   - $\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$
   - $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}, \quad \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
   - $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
   - $\cos(\pi/6) = \sin(\pi/3) = \dfrac{\sqrt{3}}{2}, \quad \cos(\pi/3) = \sin(\pi/6) = \dfrac{1}{2}, \quad \cos(\pi/4) = \sin(\pi/4) = \dfrac{\sqrt{2}}{2}$

3. Transforming polar to rectangular (Cartesian) coordinates: $x = r\cos\theta$, $y = r\sin\theta$

4. Area in polar coordinates: $dA = \dfrac{1}{2}r^2 d\theta$

5. Arc length:
   - For a parametric curve $(x(t), y(t))$: $d\ell = \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\, dt$
   - For the graph of a function $y(x)$: $d\ell = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\, dx$
   - For a polar graph $r(\theta)$: $d\ell = \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$

6. Complex exponentials: $e^{i\theta} = \cos\theta + i\sin\theta$

7. Hyperbolic functions:
   - $\cosh x = \dfrac{e^x + e^{-x}}{2}, \quad \sinh x = \dfrac{e^x - e^{-x}}{2}$
   - $\dfrac{d}{dx}\cosh x = \sinh x, \quad \dfrac{d}{dx}\sinh x = \cosh x$
   - $\cosh^2 x - \sinh^2 x = 1$

8. Geometric series: $\displaystyle\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + r^3 + \ldots = \frac{1}{1-r}$ if $|r| < 1$

9. Taylor expansions:
   - $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$
     or if you prefer...
   - $f(a+h) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}h^k = f(a) + f'(a)h + \frac{f''(a)}{2!}h^2 + \ldots$

10. Some favorite Taylor series:
    - $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
    - $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$
    - $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$
    - $\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

11. Limit comparison test: Suppose $a_k > 0$, $b_k > 0$ and $a_k/b_k \to L > 0$: then $\sum a_k$ converges if and only if $\sum b_k$ converges.

12. Alternating series test: Suppose $\{a_k\}$ is a decreasing sequence of positive numbers, converging to 0: then $\sum (-1)^k a_k$ converges.

13. Integral test, comparison test, test for divergence: you should know these off the top of your head.

14. Radius of convergence: If $a_{k+1}/a_k \to L$, then $\sum a_k x^k$ has radius of convergence $1/L$.

15. Integrating factors: To solve $y' + p(x)y = q(x)$, compute $H(x) = \int p(x)\, dx$ and multiply both sides by $e^{H(x)}$.

Now here's a list of topics you will **not** need to know for the final:

- Numerical methods for integrals and differential equations
- Remainder formula for Taylor polynomials
- Bessel functions
- Harmonic motion (though it may come up in a *bonus* question)