

Calculus II
Spring 2004
Chris Wendl

Optional Homework: Complex Numbers

1.) Solve the following two equations:

- (a) Find all complex numbers z in the form $z = r \cdot (\cos(\theta) + i \sin(\theta))$, where $r > 0$, satisfying $z^5 = 1$.
- (b) Find all solutions of $z^2 + z + 1$.

2.) Write the following complex numbers in the form $A + iB$:

- (a) $\frac{5+3i}{6-i}$.
- (b) $\frac{\pi+i}{5-i}$.
- (c) $(1+i)^8$.

3.) Find all complex numbers z so that

$$\Re(z^2) + \Im(z^2) = 0.$$

Here \Re and \Im denote the real and imaginary part, respectively.

4.) Find and sketch the sets of complex numbers defined by the following:

- (a) $\Re(z) \geq 1$.
- (b) $\Re(z) \cdot \Im(z) = 1$.
- (c) $|z| > 1$.

5.) Just using the power series expansion for $\sin(\theta)$ show that

$$\sin(-\theta) = -\sin(\theta).$$

6.) Compute the following (bring (a) into the form $A + iB$):

- (a) $\left(\frac{1}{1+\frac{1}{1+i}}\right)^2$
- (b) $|4 + 3i|$.