

Midterm 1: Formula Sheet

1. Some useful antiderivatives:

$$\begin{aligned}\int \sec^2 x \, dx &= \tan x + C & \int \csc^2 x \, dx &= -\cot x + C \\ \int \sec x \, dx &= \ln |\sec x + \tan x| + C & \int \csc x \, dx &= \ln |\csc x - \cot x| + C \\ \int \sec x \tan x \, dx &= \sec x + C & \int \csc x \cot x \, dx &= -\csc x + C\end{aligned}$$

2. Trigonometry:

- $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \csc^2 \theta$
- $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
- $\cos(\pi/6) = \sin(\pi/3) = \frac{\sqrt{3}}{2}$, $\cos(\pi/3) = \sin(\pi/6) = \frac{1}{2}$, $\cos(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$

3. Transforming polar to rectangular (Cartesian) coordinates: $x = r \cos \theta$, $y = r \sin \theta$

4. Area in polar coordinates: $dA = \frac{1}{2}r^2 d\theta$

5. Arc length:

- For a parametric curve $(x(t), y(t))$: $d\ell = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- For the graph of a function $y(x)$: $d\ell = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- For a polar graph $r(\theta)$: $d\ell = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

6. Complex exponentials: $e^{i\theta} = \cos \theta + i \sin \theta$