Calculus II
Spring 2004
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Midterm 1: Review Questions

Integration Problems

The following problems in the textbook are good for practicing the integration techniques we've covered: Pp. 512–513 #17,18, 22–28, 35, 36, 38, 42, 43.

Answers

17. $2/3$ **18.** $\frac{1}{2}\arctan\left(\frac{1}{2}\sin x\right) + C$ **22.** $\frac{3}{4}\ln\left(\frac{17}{2}\right)$

23. $-\frac{2}{3}\cos^3 x + C$ (Hint: double-angle formula) **24.** $-\frac{1}{3}e^{-3x}(3x+1) + C$

25. $\frac{1}{2}(x+1)[\ln(x+1) - 1] + C$ (Hint: $\ln(a^{1/2}) = (1/2)\ln a$. You can compute $\int \ln w \; dw$ by parts.)

26. $\ln\left(\frac{x^2}{1+x^2}\right) + C$ **27.** $-\ln|\cos x| + \frac{1}{2}\cos^2 x + C$ **28.** $-\frac{1}{2\sin^2 x} + C$

35. $\frac{2^x}{\ln 2}\left(x - \frac{1}{\ln 2}\right)$ (Hint: $2 = e^{\ln 2}$) **36.** $\frac{3}{2}x(\ln x - 1) + C$ **38.** $\frac{1}{3}(e^8 - 1)$

42. $\sqrt{x^2+2x-8} + 2\ln\left|\frac{x+1+\sqrt{x^2+2x-8}}{3}\right| + C$ **43.** $\frac{1}{2}\arcsin x - \frac{1}{2}(x+2)\sqrt{1-x^2} + C$

Thinking Questions

These have less to do with practical calculation than with understanding the theoretical underpinnings of the subject. While you will not encounter any questions quite like these on the exam (certainly nothing so open-ended), it would be a good idea to give a little time and thought to these during your studying.

1. One way to write the formula for integration by parts is

$$\int f(x)g'(x)\; dx = f(x)g(x) - \int f'(x)g(x)\; dx,$$

   or for definite integrals,

$$\int_a^b f(x)g'(x)\; dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)\; dx.$$

   How do these follow from the product rule for derivatives? In other words, pretend for a moment that you know nothing about integration by parts, but you understand the product rule perfectly well. Now how could you derive these formulas?

2. Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ be any polynomial of degree n. What would you have to do to compute $\int P(x)e^x \; dx$?

3. Suppose $G(x)$ is a function defined by the following two properties: $G(1) = 0$, and $G'(x) = e^{-x^2}$ for all x. How, in principle, could you compute the value $G(3)$, if your life depended on it? (You will not be able to get an exact answer, but an approximation is good enough.)

4. Given a parametric curve $(x(t), y(t))$, how would you compute the slope of the curve at the point $(x(t_0), y(t_0))$ for some given time t_0? Now suppose you have a polar curve $r = F(\theta)$: show that the slope of the curve at a given point $[F(\theta_0), \theta_0]$ is

$$\frac{F'(\theta_0)\sin\theta_0 + F(\theta_0)\cos\theta_0}{F'(\theta_0)\cos\theta_0 - F(\theta_0)\sin\theta_0}$$

Hint: you can always turn a polar curve into a parametric curve, and thus use the first question to answer the second. Use θ as a parameter and write x and y as functions of θ.

5. Let $z(t)$ be a function on the interval $t_0 \le t \le t_1$ with values that are *complex* numbers: its real and imaginary parts are *real*-valued functions $x(t)$ and $y(t)$, so we write

$$z(t) = x(t) + iy(t).$$

You can think of $z(t)$ as tracing out a curve in the complex plane, just as a parametric curve is traced out in the xy-plane. (So for instance, $z(t) = e^{it}$ for $t \in [0, 2\pi]$ traces out the unit circle in the complex plane.) We define the derivative of $z(t)$ in a natural way: it is the complex-valued function $z'(t) = x'(t) + iy'(t)$. Show that the length of the curve traced out by any such function $z(t)$ is always given by

$$L = \int_{t_0}^{t_1} |z'(t)| \, dt.$$

Then use this formula to compute (yet again) the circumference of a unit circle.