V63.0005.002, Spring 2002 Mathematical Thinking Chris Wendl

Midterm 1

Answer as many questions as you can. Write your solutions in the blue test booklet, showing enough work to demonstrate how you arrived at the answers. You may do the problems in any order you wish. You're also encouraged to use scrap paper, but it will not be graded.

- 1. [15 points] Expand and/or simplify each of the following. The answer should be in standard polynomial form, e.g. 2x(x+1) should be rewritten as $2x^2 + 2x$.
 - (a) $(-4x^2 + 5x 3) (x^3 + 2x^2 + x 7)$
 - (b) $(a-3)(a^2+3a-9)$
 - (c) $(p^3-3)^2-(p^3+3)^2$
- 2. [20 points] Find all solutions to each of the following equations:
 - (a) $y^2 y = 6$
 - (b) $2x^2 + 13x + 15 = 0$
 - (c) $-4t^2 + 20 = 4$
 - (d) $x^4 + 16 = 8x^2$
- 3. [18 points] Consider the line represented by the equation 3x y = 3.
 - (a) Does the point (2, -1) lie along the line?
 - (b) Does the point (2,3) lie along the line?
 - (c) What is the slope of the line?
 - (d) What is its y-intercept? (i.e. the y-coordinate of the point where the line intersects the y-axis)
 - (e) Sketch a graph of the line, labelling the coordinates of any two points that lie along it (choose whichever points you find convenient).
 - (f) Draw a new line on your graph: one that intersects the first one perpendicularly at its y-intercept. Now write down an equation for this new line.
- 4. [18 points] Let $f(x) = x^2 2x$ and g(x) = 5x 4. Find each of the following (simplify where appropriate):
 - (a) $(g \circ f)(x)$
 - (b) (f g)(x)
 - (c) $g^{-1}(x)$
- 5. [14 points] Mildred makes a long distance "person-to-person" call to San Francisco, California. The telephone company charges 41 cents for the first minute and 32 cents for each additional minute. Because the call is person-to-person, there is also a service charge of \$3.00 If the cost of the call is \$6.29, how many minutes did the call last?
- 6. [15 points] The following statistics are made up, but pretend for now that they're true. Census figures indicate that the population of New York City in 1990 was about 8.5 million people, while in 2000 it had gone up to 8.7 million. If we assume a steady rate of increase during the intervening years, we can estimate the population in any year between 1990 and 2000. "Steady rate of increase" means we assume we can describe the population as a function of time which is linear, i.e. its graph is a line.
 - (a) Using whatever notation seems sensible to you, write down an expression for the population as a function of time. (Be sure to specify the meaning of any variables you use.)
 - (b) Assuming your answer to part (a) is right, what was the population of New York City in 1993?