HIGHER STRUCTURE: EXERCISE 4

DINGYU YANG

This is the 1.4 version and ready. Do not panic at the length of this Exercise Sheet, it is so partly because it contains some lecture notes for people not present in some of the recent lectures and it also contains a fairly long optional Exercise. Hopefully it takes less time to read/work through them than I typed them up. The lecture this Friday will cover the Weinstein neighborhood theorem for Lagrangians and the index theorem for the virtual dimension of the moduli space $\mathcal{M}_{k+1}(\beta)$, so that a large subgroup of the audience who are going to the Aachen for a 2-day conference can safely miss it. The exercise session this Friday will cover (some of) the following:

Exercise 1: In Lecture 6, we covered an instance of a formality criterion. In this guided exercise, we prove the formality for Kähler manifolds. Here the full structure is CDGA (commutative differential graded algebra) but we will ignore the commutative aspect which is not featured in the proof below anyway (the commutative aspect is important for discussion on rational/real homotopy types).

Show that for a Kähler manifold (X, h), the de Rham cochain complex is quasi-isomorphic to the induced de Rham cohomology as A_∞ algebras, a DGA qusi-isomorphism for short. The proof consists of the following steps. Here all products involved below are wedge products (with your favorite sign convention or without signs) with higher products vanishing.

(1) (dd^c-Lemma for Kähler manifold) Let (X, h) be a closed Kähler manifold, and let $\alpha \in \Omega^k$ such that $d\alpha = 0$ and $\alpha = d^c\gamma$. Then there exists $\beta \in \Omega^{k-2}(X)$ such that $\alpha = dd^c\beta$. Here $d^c := -i(\partial - \bar{\partial})$, where ∂ and $\bar{\partial}$ are $(1,0)$ and $(0,1)$-parts of d respectively.
(2) Let $\Omega^{k;c}(X) := \{\alpha \in \Omega^k(X) \mid d^c\alpha = 0\}$ and $d : \Omega^{k;c}(X) \to \Omega^{k+1;c}(X)$. Show that the inclusion $\iota : (\Omega^{*;c}(X), d) \to (\Omega^*(X), d)$ is a DGA quasi-isomorphism.
(3) Define $H^k_{d^c} := H^k(\Omega^*(X), d^c)$, and we have $d : H^k_{d^c}(X) \to H^{k+1}_{d^c}(X)$ and d acts as $d = 0$. Show that the natural projection $p : (\Omega^{*;c}(X), d) \to (H^*_{d^c}(X), d = 0)$ is a DGA quasi-isomorphism.
(4) $(\Omega^*(X), d)$ is DGA quasi-isomorphic to $(H^*_{d^c}(X), 0)$ (which is DGA isomorphic to the cohomology algebra of the former), which means $(\Omega^*(X), d)$ is formal.

Exercise 2: In the lecture, we used the formula for Hochschild cochain complex for A_∞ algebras. As an exercise, find a reference to extract the definition and basic properties for the Hochschild cochain complex for an A-bimodule M, where A is a (graded) algebra.

Date: November 13, 2019.

Exercise 3: In the lecture, for a Fredholm section $f : B \to E$ of a Banach bundle over a Banach manifold and $p \in f^{-1}(0)$, choose U such that $p \in U \overset{\text{open}}{\subset} B$ and L_p finite dimensional such that the span of the image $(\nabla f)_p : T_pB \to E_p$ and L_p is E_p and L_p is extended to a vector bundle L (as a subbundle of E) over U. By construction the quotient Fredholm section $f|_U/L : U \to E|_U/L$ is transverse to the zero section at p, and by open condition of tranversality, it is transverse to the zero section over $\tilde{U}$ which is open neighborhood of p in U. Define $V := (f|_{\tilde{U}}/L|_{\tilde{U}})^{-1}(0)$ and by construction $s := f|_V$ maps from V to $\mathcal{E} := L|_V$. Another equally good alternative choice to $L|_{\tilde{U}}$ (from which we can extract the data $\tilde{U}$) is the following: For any open neighborhood U' of p in $\tilde{U}$ and any finite rank L' over U' (subbundle of E) for which $L|_{U'}$ is a subbundle, we can define $f|_{U'}/L'$ which is transverse to the zero section along and near $V \cap U'$ and thus so in an open neighborhood $\tilde{U}'$ of $V \cap U'$ in U'. We can also use $L'|_{\tilde{U}'}$ to get a finite dimensional reduction $s' : V' \to \mathcal{E}'$.

Actually we have an embedding of $s|_{V\cap V'}$ into s, which means a bundle embedding $\mathcal{E}|_{V\cap V'} \to \mathcal{E}'$ (covering embedding $V \cap V' \to V'$) that intertwines these two sections, which suggests the form of coordinate change between Kuranishi charts should look like (I will cover this in more detail after I have explained more geometric aspect of moduli spaces). One important property for this embedding is the following *tangent bundle condition*:

As $\nabla s'|_{T_x(V\cap V')} = \nabla s|_{T_x(V\cap V')}$ for any $x \in V \cap V'$, we can define the induced normal linearization

$$(\nabla^N s')_x : T_xV'/T_x(V \cap V') \to \mathcal{E}'_x/\mathcal{E}_x,$$

which is well-defined independent of the choice of ∇ for any $x \in s^{-1}(0)$. Show that this is an isomorphism between vector spaces by construction for any $x \in s^{-1}(0)$.

Exercise 4: For a submersion $e : \mathcal{M} \to L$ mapping from a Kuranishi structure and a de Rham form η on $\mathcal{M}$ and a de Rham form ζ on L, we have the projection formula: $\zeta \wedge e_!^{\mathfrak{c}}\eta = e_!^{\mathfrak{c}}(e^*\zeta \wedge \eta)$, where $\mathfrak{c}$ is some choice involved to define integration along fibers for Kuranishi structure $\mathcal{M}$ as discussed in the lecture. (See the end of this document for a semi-explanation of integration along the fibers in the Kuranishi structure setting, for the convenience of those who was not here during this Tuesday's lecture.) Show that for $\partial_{k_1,\beta_1,i}\mathcal{M}_{k+1}(\beta) := \mathcal{M}_{k_1+1}(\beta_1)_{ev_0} \times_{ev_i} \mathcal{M}_{k_2+1}(\beta_2)$ (here the index is sufficient as k_2 and β_2 will then be determined) with evaluation maps

$$(ev_1^2, \cdots, ev_{i-1}^2, ev_1^1, \cdots, ev_{k_1}^1, ev_{i+1}^2, \cdots, ev_{k_2}^2)$$

to pullback forms and with ev_0^2 to integrate along fibers, and with choices $\partial_{k_1,\beta_1,i}\mathfrak{c} = \mathfrak{c}_1 \ {}_{ev_0}\times_{ev_i} \mathfrak{c}_2$ (where $\mathfrak{c}_1$ and $\mathfrak{c}_2$ are the choices for Kuranishi structures $\mathcal{M}_{k_1+1}(\beta_1)$ and $\mathcal{M}_{k_2+1}(\beta_2)$ to define integration along fibers of ev_0^1 and ev_0^2 respectively), we have the following identities (modulo signs):

$$\begin{aligned}
&(ev_0)_!^{\partial_{k_1,\beta_1,i}\mathfrak{c}}(ev_1^*\eta_1 \wedge \cdots \wedge ev_k^*\eta_k)\\
=&(ev_0^2)_!^{\mathfrak{c}_2}\left((ev_1^2)^*\eta_1 \wedge \cdots \wedge (ev_i^2)^*\left((ev_0^1)_!^{\mathfrak{c}_1}\left((ev_1^1)^*\eta_i \wedge \cdots \wedge (ev_{k_1}^1)^*\eta_{i+k_1-1}\right)\right) \wedge \cdots \wedge (ev_{k_2}^2)^*\eta_k\right)\\
=&m_{k_2;\beta_2}(\eta_1, \cdots, m_{k_1;\beta_1}(\eta_i, \cdots, \eta_{i+k_1-1}), \cdots, \eta_k).
\end{aligned}$$

Exercise 5: The following is a good fun exercise (especially suited for riding a train for extended hours), but optional. I will write some text leading up to it (which is covered towards the end of lecture as well). Recall in the lecture, we have the Stokes-type formula

$$d(e_!^{\mathfrak{c}}\eta) = e_!^{\mathfrak{c}}(d\eta) + (-1)^{\dim \mathcal{M}+\deg \eta}(\partial e)_!^{\partial \mathfrak{c}}\big((\iota_{\partial \mathcal{M}})^*\eta\big),$$

where we continue using the notation from the start of Exercise 4, and the last term is integration along fibers of $\partial e : \partial \mathcal{M} \to L$ which is induced from $e : \mathcal{M} \to L$ by taking codimension-1 closed boundary strata, using the induced choice $\partial \mathfrak{c}$, and $\iota_{\partial \mathcal{M}} : \partial \mathcal{M} \to \mathcal{M}$ is the immersion of the 'inclusion' of the boundary (recall that the corner strata is equipped with the data remembers which iterated higher strata it comes from, thus with multiplicities and hence an immersion only, even within the same connected component). This when applied to

$$(e : \mathcal{M} \to L, \eta) = (ev_0 : \mathcal{M}_{k+1}(\beta) \to L, ev_1^*\eta_1 \wedge \cdots \wedge ev_k^*\eta_k)$$

together with the previous Exercise yields the A_∞ relation which also explains why we include $m_{1,0} = (-1)^{n+1+\deg \eta}$ term in m_1. In fact, this is only true provided countable $\mathfrak{c} := \mathfrak{c}_{\mathcal{M}_{k+1}(\beta)}$'s can be chosen all at once to be compatible, which is very hard to do and is only done as a homotopy limit in the literature (various finite amounts of compatible choices can be compared in a controlled way via chain homotopies) and here we suppress this subtle point.

Note here we have m_0 term in the A_∞ algebra structure $\{m_k\}_{k\geq 0}$. The operation

$$m_1 = \sum_{\beta \in H_{2;\mathrm{holo}}(X,L;\mathbb{Z})} m_{1;\beta} T^{E(\beta)}$$

is the natural candidate for the 'differential' for the Lagrangian (Hamiltonian) Floer homology with 0 Hamiltonian based on de Rham cochain model in the motivating example, where $H_{2;\mathrm{holo}}(X,L;\mathbb{Z})$ denotes the subspace of $H_2(X,L;\mathbb{Z})$ whose elements are representable by nodal holomorphic disks. And the obstruction for this being a square 0 differential is precisely explained by the second identity (the first identity says $m_0(1)$ is m_1-closed) of the he A_∞ relation,

$$m_1 \circ m_1 = -m_2 \circ_1 (m_0(1)) - (-1)^{\deg \eta - 1} m_2 \circ_2 (m_0(1)).$$

Namely, have disk bubbling on either side of boundary between 0-th and 1st boundary marked points. This is mentioned towards the end in the lecture, but convince yourself that $m_0(1) := \sum_{\beta \in H_{2;\mathrm{holo}}(X,L;\mathbb{Z})} \big((ev_0^{\mathfrak{c}_{\mathcal{M}_1(\beta)}})_! 1\big) T^{E(\beta)} \in \Omega^*(L)\hat{\otimes}\Lambda_0$, not just in Λ_0.

There are increasingly flexible ways to cook up squaring 0 differentials from this 'curved' A_∞ algebra structure, all of which will be covered in the lecture in the near future. The following is one and its geometric explanation is the content of this exercise. For any $b \in \Omega^1(L)\hat{\otimes}\Lambda_+$, here completed tensor with the maximal ideal $\Lambda_+ \subset \Lambda_0$ is for convergence reason (however, we will cover an improvement in the upcoming lecture). We can define

$$m_k^b(x_1, \cdots, x_k) := \sum_{n_0 \geq 0, n_1 \geq 0, \cdots n_k \geq 0} m_{k+\sum_{i=0}^k n_i}\big(b^{\otimes n_0}, x_1, b^{\otimes n_1}, x_2, \cdots, x_k, b^{\otimes n_k}\big).$$

Then $\{m_k^b\}_{k\geq 0}$ is another curved A_∞ algebra. b such that $m_0^b(1)=0$ is called a *bounding cochain*, or a *Maurer-Cartan element*. To be more explicit, the requirement

$$m_0^b(1) = m_0(1) + m_1(b) + m_2(b,b) + \cdots + m_k(b^{\otimes k}) + \cdots = 0.$$

The following obstruction-theoretic interpretation explains why b is called a bounding cochain and is borrowed (and slightly reformulated) from Ohta's survey (also in FOOO book 1 in some way) but please attempt first before consulting:

(1) The first identity in A_∞ relation says $m_0(1)$ is m_1-closed, thus its first term $o_1 := (ev_0^{\mathfrak{c}_{\mathcal{M}_1(\beta_1)}})_!(1)$ is in particular d-closed for the de Rham differential d. Here β_1 is the class with the minimal positive area. If it is exact, then we can find η_1 that bounds it: $d\eta_1 = (-1)^{n+1}o_1$ (one can ignore the sign but this is consistent with $m_{1,0}$).
(2) Countably order $H_+ := H_{2;\text{holo}}(X,L;\mathbb{Z})\backslash\{0\}$ into $\{\beta_k\}_{k\geq 1}$ such that if $i<j$ then $E(\beta_i)\leq E(\beta_j)$.
(3) Suppose we have defined d-closed forms o_j for $1\leq j\leq k-1$ and they are all exact: $o_1 = 0,\cdots,o_{k-1}=0$ and we have inductively chosen bounding cochains η_j such that $d\eta_j = (-1)^{n+1}o_j$ for $1\leq j\leq k-1$.
(4) For $I := \{i_1,\cdots,i_m\}\subset \underline{k-1} := \{1,\cdots,k-1\}$, denote

$$\mathcal{M}_1(\beta_k;I) = \mathcal{M}_{m+1}(\beta_k - \beta_{i-1} - \cdots - \beta_{i_m})$$

and $\mathfrak{c}_{k;I} := \mathfrak{c}_{\mathcal{M}_1(\beta_k;I)}$. Define

$$o_k := \sum_{m=0,1,\cdots,\ I\subset \underline{k-1},\ \beta-\sum_{j=1}^m \beta_{i_j}\in H_+} (-1)^{\bullet}\frac{1}{m!}(ev_0)_!^{\mathfrak{c}_{k;I}}(ev_1^*\eta_{i_1}\wedge\cdots\wedge ev_m^*\eta_{i_m}),$$

with η_j previous the previous item.

Use the previous item and the Stokes and projection formulae to show o_k is closed.
(5) If o_k is exact, we can choose bounding chain η_k such that $d\eta_k = (-1)^{n+1}o_k$.
(6) Show that by construction,

$$b := \sum_{i\geq 1}\eta_i T^{E(\beta_i)}$$

satisfies $m_0^b := \sum_{k\geq 0} m_k(b^{\otimes k}) = 0$.

Exercise 6: (This gives a bridge between the 'integration along fibers' approach with the usual index-0/rigid count of moduli spaces of nodal disks with appropriate incidence conditions) Bear in mind below, things cannot descend to the homology level because we are dealing with moduli spaces with codimension 1 boundary, things are well-defined as part of chain level structure in the sense of higher structure.

By Thom, any class $[a]\in H_i(L;\mathbb{R})$ can be represented by an embedded submanifold $S_a\subset L$ with real weight $w_a\in\mathbb{R}$. $[a]=w_a[S_a]$. For any submanifold, we can use tubular neighborhood and Thom form to construct a PD representative ν_a. Choose cycles $a_1,\cdots,a_k$ and associated dual closed forms ν_{a_i}. We look at $m_{k,\beta}(\nu_{a_1},\cdots,\nu_{a_k}) = (ev_0)_!^{\mathfrak{c}}(ev_1^*\nu_{a_1}\wedge\cdots\wedge ev_k^*\nu_{a_k})$ where we integrate along fibers of $ev_0 : \mathcal{M}_{k+1}(\beta)\to L$ using some necessary choice $\mathfrak{c}$. The output is a d-form where $d = \deg\nu_{a_1}+\cdots+\deg\nu_{a_k}+2-k-\mu(\beta)$. One knows what this form is by pairing with all d-cycle S_a. The following is only chain level identity depending on choices $\mathfrak{c}$

because $\mathcal{M}_{k+1}(\beta)$ has boundary and corners (besides the transversality issue) and extracting a number by intersecting (a map from) a 'manifold' with boundary and corners will not be in general well-defined independent of choices.

We have

$$\begin{aligned}
&\langle (ev_0)_!^{\mathfrak{c}}(ev_1^*\nu_{a_1} \wedge \cdots \wedge ev_k^*\nu_{a_k}), S_a\rangle \\
=&\langle \nu_a \wedge (ev_0)_!^{\mathfrak{c}}(ev_1^*\nu_{a_1} \wedge \cdots \wedge ev_k^*\nu_{a_k}), L\rangle \\
=&\langle ev_0^*\nu_a \wedge ev_1^*\nu_{a_1} \wedge \cdots \wedge ev_k^*\nu_{a_k}, \mathcal{M}_{k+1}(\beta)\rangle \\
=&\langle pr_0^*\nu_a \wedge pr_1^*\nu_{a_1} \wedge \cdots \wedge pr_k^*\nu_{a_k}, (ev_0 \times ev_1 \times \cdots ev_k)_*\mathcal{M}_{k+1}(\beta)\rangle \\
=&(S_a \times S_{a_1} \times \cdots \times S_{a_k}) \cap (ev_0 \times ev_1 \times \cdots \times ev_k)_*\mathcal{M}_{k+1}(\beta) \\
=&\{[\Sigma, (z_0, z_1, \cdots, z_k), u] \in \mathcal{M}_{k+1}(\beta) \mid u(z_0) \in S_a, u(z_1) \in S_{a_1}, \cdots u(z_k) \in S_{a_k}\}.
\end{aligned}$$

Try to (heuristically) justify each line (each setting might require different transversality).

Recap[1] 7: Integration along fibers for a submersion from a Kuranishi structure (locally or assuming only one Kuranishi chart, more global treatment is due in one of the upcoming lectures):

For integration along fibers for map $e : \mathcal{M} \to L$ from a Kuranishi structure. We unpeal this mysterious definition through several layers in increasing generality.

(1) If e is a proper submersion from a compact manifold $\mathcal{M}$, then by Ehresmann fibration theorem, e is a fiber bundle over L. For a differential form η on M, we can define $e_!\eta$ which will be a smooth differential form on L. Covering L by partition of unity $\{\lambda_\alpha\}_{\alpha\in\mathcal{A}}$ subordinate to a cover of L by trivializing neighborhoods $\{O_\alpha\}$: $\mathcal{M}|_{O_\alpha}$ is bundle diffeomorphic to $O_\alpha \times F$ via Ψ_α. For a differential η on $\mathcal{M}$, $(\Psi_\alpha^{-1})^*(\lambda_\alpha\eta)$ is a compactly supported differential form on $O_\alpha \times F$ and can be written as a finite sum $\sum_{I,J} f_{IJ}(x,y)\gamma_I \wedge \delta_J$ where γ_I and δ_J are differential forms on O_α and F respectively. We define $(pr_1)_!(\sum_{I,J} f_{IJ}\gamma_I \wedge \delta_J) := \sum_{I,J}(\int_F f_{IJ}(x,y)\delta_J)\gamma_I$, (there are different sign conventions, this way conforms with Bott-Tu yields projection formula without signs), where the integral is 0, unless δ_J is a top degree form on F. One defines $e_!\eta := \sum_\alpha (pr_1)_!\big((\Psi_\alpha^{-1})^*(\lambda_\alpha\eta)\big)$, this definition can be checked to be independent of choices.

(2) If $e : \mathcal{M} \to L$ maps from a not necessarily compact space $\mathcal{M}$, and η is a compactly supported form on $\mathcal{M}$, one can define $e_!\eta$ using local models for submersion and partition of unity similarly as above, and one shows it is well-defined independent of choices.

(3) If $\mathcal{M}$ is a compact manifold, and $e : \mathcal{M} \to L$ is not necessarily a submersion and η is a smooth differential form on $\mathcal{M}$. One can define thickening $\mathbf{e} : \mathcal{M} \times W \to L$ where W is open neighborhood of 0 of some vector space N, such that $\mathbf{e}(\cdot, 0) = e$ and $\mathbf{e}$ is a submersion. Choose a top degree form ν_W in N compactly supported in W such that $\int_N \nu_W = 1$. Then one defines

$$e_!^{\mathfrak{c}}\eta := \mathbf{e}_!(pr_1^*\eta \wedge pr_2^*\nu_W)$$

where $\mathfrak{c} := (\mathbf{e}, \nu_W)$ keeps track of the choice made. The RHS is defined using item (2). Here we cannot expect independence of choices, but for another choice

[1] For the dual purpose of Exercise 4 and providing part of the lecture note of this Tuesday without extra delay. The note below slightly expands the lecture material.

$\mathfrak{c}' := (\mathbf{e}' : \mathcal{M} \times W' \to L, \nu_{W'})$ such that there exists a linear projection $pr_{W'W} : W' \to W$ and $\mathbf{e}' = \mathbf{e} \circ (\mathrm{Id}_{\mathcal{M}} \times pr_{W'W})$ with $(pr_{W'W})_! \nu_{W'} = \nu_W$, $e_!^{\mathfrak{c}'} \eta = e_!^{\mathfrak{c}} \eta$. So this generates an equivalence relation among choices, where equivalent choices give rise to the same output (this is due to the usual projection formula).

(4) If $\mathcal{M}$ is the compact zero set of a section s in a finite dimensional bundle $\mathcal{E} \to V$ which is trivial $\mathcal{E} := V \times F$ and $e_V : V \to L$ is a submersion with $e_V \circ \iota_{\mathcal{M}} = e$. Then the tuple (s, e_V) is called a submersion $e : \mathcal{M} \to L$ from a global Kuranishi chart for $\mathcal{M}$. We can find $W \subset N$ such that $\mathbf{s} : V \times W \to pr_1^* \mathcal{E}$ has the properties that $\mathbf{s}(\cdot, 0) = s$ and $\mathbf{s}$ is transverse to the zero section in $pr_1^* \mathcal{E}$ and denoting $\mathbf{e} := e \circ pr_1$, $\mathbf{e}|_{\mathbf{s}^{-1}(0)}$ restricted to the manifold $\mathbf{s}^{-1}(0)$ is a submersion to L. Choose ν_W as in item (3). Define

$$e_!^{\mathfrak{c}} \eta := (\mathbf{e}|_{\mathbf{s}^{-1}(0)})_! \big(\iota^*_{\mathbf{s}^{-1}(0)} (pr_1^* \eta \wedge pr_2^* \nu_W)\big),$$

where the RHS is defined due to item (2) and $\mathfrak{c} = (\mathbf{s}, \nu_W)$.

A choice for such $\mathbf{s}$ would be $\mathbf{s}_F : V \times F \to pr_1^*(\mathcal{E})$, $(x, v) \mapsto s(x) + v^2$. We denote this choice by $\mathfrak{c}_F := (\mathbf{s}_F, \nu_F)$.

(5) Same as item (4) except $e_V : V \to L$ is not submersion, one can always add a neighborhood of 0 in a finite dimensional space of vector fields on L to W and $\mathbf{e}$ on those extra directions it would be exponential map on L. The rest is the same.

(6) If $\mathcal{M}$ can only be locally covered by $s^{-1}(0)$ for $s : V \to \mathcal{E}$ with coordinate changes between charts in the form of Exercise 3. This directly goes deep into the global Kuranishi structure alluded to in the lecture, which we postpone, but we give a feeling/intuition that if we have embedding s to s' intertwined by embedding $\mathcal{E} = V \times F \to \mathcal{E}' = V' \times F'$ covering $V \to V'$ (where the fiber embedding is independent of base coordinates, which is not most general form of bundle mebdding but achievable in practice, general version also works by tweaking below), where embeddings are notationally inclusions, we can choose $W_{V'V}$ a open neighborhood of V in V' and a submersion $\pi_{V'V} : W_{V'V} \to V$, a linear projection $\pi_{F'F} : F' \to F$ and ν_F and $\nu_{F'}$ top form compactly supported and integrating to 1 on F and F' respectively such that $(\pi_{F'F})_! \nu_{F'} = \nu_F$. Choose a Thom form $Th := Th_{W_{V'V}}$ for $W_{V'V}$ as a bundle over V'. Denote $\tilde{\mathbf{s}}' = \mathbf{s}_F \circ (pr_{V'V} \times pr_{F'F})$ and $\tilde{\nu} := pr_1^* Th \wedge pr_2^* \nu_{F'}$ which will play the role of $pr_2^* \nu_{F'}$; and denote $\tilde{\mathfrak{c}} := (\tilde{\mathbf{s}}', \tilde{\nu})$. Then we have

$$e_!^{\mathfrak{c}_F} \eta = e_!^{\tilde{\mathfrak{c}}} \eta,$$

(the LHS happens for the chart s and the RHS uses the chart s') which together with a partition of unity and equivalence notion in item (3) will help patch local choices together into a global operation of the integration along fibers to yield a smooth differential form on L. We will cover this in the lecture later.

Feel free to drop by my office if you have any questions or want to discuss.

[2] This is a bit analogous to put Teichmüller slice to the domain to ensure the linearized tangent CR operator to be surjective.