

FORSCHUNGSSEMINAR ALGEBRA UND ZAHLENTHEORIE

COMPLETELY FAITHFUL MODULES OVER IWASAWA ALGEBRAS

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I would like to talk about two of my results in connection with noncommutative Iwasawa theory and the GL_2 conjectures for elliptic curves without complex multiplication [2].

1. We have a p -adic analytic group of the form $G = H \times \mathbb{Z}_p$ such that the corresponding Lie algebra of H is split semisimple. The reason we have groups of this form is in connection with elliptic curves. Let us denote the Iwasawa algebra of a p -adic analytic group G by Λ_G . Consider the full subcategory $\mathfrak{N}_H(G)$ of those finitely generated Λ_G -modules (denote this category by $\text{mod}(\Lambda_G)$) that also are finitely generated over Λ_H . Moreover we denote by \mathcal{C} the subcategory of pseudonull modules. This is a Serre subcategory, so we have a quotient category and a quotient functor:

$$q : \text{mod}(\Lambda_G) \rightarrow \text{mod}(\Lambda_G)/\mathcal{C}$$

We define the annihilator of an object $\mathcal{M} \in \text{mod}(\Lambda_G)/\mathcal{C}$ in the following way:

$$\text{Ann}(\mathcal{M}) := \sum \{ \text{ann}_{\Lambda_G}(M) : q(M) \simeq \mathcal{M} \}$$

The object is called completely faithful if $\text{Ann}(\mathcal{N}) = 0$ for any subquotient object of \mathcal{M} . Completely faithful objects are central in the GL_2 conjectures and in the theory of noncommutative Iwasawa theory in general. The first result is that, if two modules M, N in $\mathfrak{N}_H(G)$ have the same class in the Grothendieck group of $\mathfrak{N}_H(G)$, i.e. in $K_0(\mathfrak{N}_H(G))$ and $q(M)$ is completely faithful then $q(N)$ is completely faithful.

2. For groups of the form above, Konstantin Ardakov proved in [1] a Theorem that characterizes the completely faithful property. I generalized this theorem to a more general class of p -adic analytic groups and also computed the reflexive ideals of the Iwasawa algebra in that setting.

REFERENCES

- [1] K. Ardakov, Centres of Skewfields and completely faithful Iwasawa modules. *J. Inst. Math. Jussieu* 7 (2008).
- [2] J. Coates, T. Fukaya, K. Kato, R. Sujatha, O. Venjakob, The GL_2 main conjecture for elliptic curves without complex multiplication, *Publ. Math. IHES* **101** (2005), 163-208.